An Analysis of the Early Assessment Program (EAP) Assessments for Algebra II and Summative High School Mathematics

Conducted by Achieve on behalf of the California Diploma Project (ADP) and Policy Analysis for California Education (PACE)

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Achieve

Created in 1996 by the nation's governors and corporate leaders, Achieve is an independent, bipartisan, non-profit education reform organization that helps states raise academic standards and graduation requirements, improve assessments and strengthen accountability. To make college and career readiness a priority in the states, in 2005, Achieve launched the American Diploma Project (ADP) Network. Starting with only a handful of states, the Network has now grown to include 35 states educating nearly 85 percent of all U.S. public school students. California joined the ADP Network in 2008, bringing together the Governor, the Superintendent of Public Instruction, the leaders of the state's four higher education sectors, and the business community.

PACE

PACE (Policy Analysis for California Education) is an independent, non-partisan research center based at the University of California – Berkeley, the University of Southern California, and Stanford University. PACE seeks to define and sustain a long-term strategy for comprehensive policy reform and continuous improvement in performance at all levels of California's education system, from early childhood to post-secondary education and training. PACE bridges the gap between research and policy, working with scholars from California's leading universities and with state and local policymakers to increase the impact of academic research on educational policy in California. The California Diploma Project is managed by PACE and is funded by the Bill & Melinda Gates Foundation, William and Flora Hewlett Foundation, and the James Irvine Foundation.

The California Diploma Project

The American Diploma Project (ADP) works with states across the country to bring value to the high school diploma by raising the rigor of high school standards, assessments and curriculum and aligning them to the demands of postsecondary education and careers. The California Diploma Project brings together the Governor and Superintendent of Public Instruction with the leaders of the state's four higher education sectors and business community representatives to work together to expand the number of young people who enroll and succeed in post-secondary education and training.

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Executive Summary

Leaders from the California State University System (CSU) and the California Department of Education have collaborated in the creation of a unique approach to college-ready assessment. The Early Assessment Program (EAP) provides students with an "early indicator" of their level of college preparation so that they might increase their knowledge and skill while still in high school and avoid failure and remediation in their entry-level college experience. This report provides an analysis of that assessment.

Achieve and the California Diploma Project

The California Diploma Project (CDP) sought the assistance of Achieve in order to determine the degree to which the EAP tests in Algebra II, Summative High School Mathematics (Summative Mathematics), and English provide an assessment of the level of college readiness required for entry-level coursework. For the purpose of this analysis of these EAP assessments, Achieve considered the California standards and additional expectations targeted by the CSU faculty as the benchmarks for college readiness—that is, as indicators of the level of preparation needed for entry into credit bearing coursework in English and mathematics on the CSU campuses.

The Early Assessment Program (EAP)

The EAP serves as a college readiness signal for students while still in high school, enabling them to better prepare for college in their senior year. It is voluntary and only available to students in the 11th grade. The EAP is a combination of select items from the California Standards Test (CST)—in Algebra II, Summative Mathematics, and Grade 11 English-Language Arts (ELA)—and a set of 15 augmented items developed by faculty at CSU. In English, the EAP also includes a direct writing assessment—the EAP Essay.

While voluntary, the number of students taking the EAP has increased over the last five years. It is important to note that while many students are in English classes during their 11th grade year and are eligible to take the English EAP, the Algebra II and Summative Mathematics EAP are limited to students who are enrolled in or have completed Algebra II. Current data follow on those 11th grade students who participated in the EAP.

How many 11th graders participated in the 2009 CST and the EAP?

Subject	Total 11th graders taking the CST	Total 11th graders taking the EAP
English	447,742 96%	366,925 79%
Mathematics	220,605 47%	169,473 36%

Out of 466,303 students enrolled in 11th grade in 2009, 96% of all students took the CST in English and 79% took the 15 augmented items and the Direct Writing Assessment for the EAP in English. In 2009, 47% of all 11th graders took the CSTs in Algebra II or Summative Mathematics and 36% took the 15 augmented items on either one of these tests for the Algebra II or Summative Mathematics EAP. In mathematics, the remaining 53% of all 11th graders did not have the option of taking the EAP. Current overall performance data follow on those 11th grade students who participated in the EAP.

How do 11th graders perform on the EAP?

Subject	Ready for College	Conditionally Ready for College	Did not Demonstrate Readiness
English	16%	N/A	83%
Algebra II	5%	20%	74%
Summative Mathematics	21%	67%	12%

Of the 79% of all 11th graders who took the EAP in English – 16% scored ready for college and 83% did not demonstrate readiness. Of the 36% of all 11th graders who took the EAP in Algebra II, 5% scored ready for college, 20% conditional, and 74% did not demonstrate readiness. In Summative Mathematics, 21% scored ready for college, 67% conditional, and 12% did not demonstrate readiness.

Achieve Analysis

Achieve convened a group of content experts in English and mathematics to conduct the analysis of the EAP English, Algebra II, and Summative Mathematics assessments. Review panels were guided by the criteria set forth in the Achieve Assessment to Standards (ATS) Protocol that include content centrality, performance centrality, source of challenge, and level of cognitive demand.

One representative form for each assessment—English Language Arts, Algebra II, and Summative High School Mathematics—was reviewed. Due to state policy restrictions, Achieve was unable to review full, intact forms of the CST assessments. This analysis therefore, should not be interpreted as a review of the CST but rather as a review of the EAP assessments that are comprised of a subset of CST items and augmented items developed by the CSU faculty. The CST items as well as the augmented items and writing samples were submitted to Achieve by the Educational Testing Service (ETS), the organization responsible for developing the CST, as representative of the items used by CSU to arrive at a student's EAP college readiness score.

Findings

1. ***The EAP assessments address essential college-ready content identified by CSU faculty for entering credit-bearing coursework.***

The CSU faculty identified college readiness skills required for students' success in entry-level coursework in English and mathematics. These college readiness skills include California standards as well as CSU related ELM and EPT assessed content. In mathematics, reviewers found that the majority of the items that were mapped to the California Algebra II and Summative Mathematics standards and the CSU ELM specifications consistently match the intent of the standard. In English, similar results were found in reading, the EAP Augmented items, and the EAP Essay assessment of direct writing. Only the indirect writing items did not satisfy the expectation of the standards, a difficulty typically faced by large-scale assessments that test instructional content standards requiring writing performance with multiple-choice items. The remaining majority of the EAP items target quite clearly the content described in these college readiness standards. These assessments can be depended upon to present an effective picture of student readiness for entry-level coursework on a CSU campus.

2. ***The EAP augmentations provide rigor and contribute authentic college readiness tasks to the assessment of students' postsecondary preparation.***

In mathematics, a particular strength of the augmentation item set is that many items require students to make connections across and/or within mathematical strands. These items tend to be interesting and mathematically rich and can be used as models for developing future items. It is noteworthy that the level of cognitive demand of these items raises the overall cognitive demand and rigor of the Algebra II and Summative Mathematics EAP.

In English, the EAP Essay that was designed by CSU faculty as a measure of direct writing is particularly noteworthy. Students are asked to demonstrate the depth of their understanding of the reading passage through their writing. The EAP Essay not only assesses key content and performance expectations called for in the California standards, but it also provides an authentic college-ready measure of a student's ability to critique, analyze, construct arguments, and support ideas with relevant examples. In addition, the reading passages included on the EAP were judged by Achieve as representing a rigorous set of texts that are fair examples of the types of materials students should be able to read to demonstrate college readiness.

3. ***Targeted improvements in the Algebra II, Summative Mathematics, and English EAP would increase the balance and rigor of the items used to assess college readiness.***

In mathematics, reviewers noted the procedural nature of many of the multiple-choice items. The inclusion/development of additional selected-response items that resemble the more cognitively demanding augmentation items, would ensure the assessment of the kinds of higher order skills students need to be prepared for college. The inclusion of application problems from the social sciences or sciences would enable students to demonstrate their abilities to use mathematics to model relevant situations with functions. The addition of a

limited number of high quality constructed-response, or open-ended, items could also increase cognitive demand. Such items can be expensive, given costs of scoring. However, these items would allow teachers to see first-hand how students approach solving problems, whether they can create graphs on their own, whether they can create conjectures or justifications on their own, and whether they can explain their reasoning.

In addition, reviewers noted a few areas of college-ready mathematics content that are in the California standards but were omitted from the Algebra II and Summative Mathematics EAP including probability and statistics and properties of logarithms and the inverse relationship between exponents and logarithms. Suggestions have been made for adjustments that would allow inclusion of these areas.

In English, reviewers found that the reading passages were complex and closely approximate the level of texts that students might encounter in college. In some instances, however, the passage was underutilized as the test-taker did not need to refer to the reading to answer the question posed. Achieve reviewers suggested that to the greatest extent possible, items that are selected for the EAP require that students read the passage in order to answer the item, providing a more accurate picture of their ability to derive meaning.

In addition, while indirect items in writing assessed students' ability to identify the correct use of conventions and grammar, reviewers found the EAP Essay provided a more direct and authentic demonstration of the writing process.

Recommendations

1. ***Since the EAP is a good representation of college readiness, use of the test could reasonably be expanded beyond the CSU.***

Overall Achieve reviewers found the test aligns well with the expectations the CSU faculty have identified as essential for entry into credit-bearing coursework. Current plans in the state call for the expansion of the EAP to selected community colleges who are piloting this initiative. Achieve supports this expansion beyond the CSU campuses and recognizes the significant impact this initiative has had for students. Steady growth in the rate of student participation and improved performance provide testimony to its success. The students of the state of California are the ultimate beneficiaries of this "early indicator" program and are given the opportunity to prepare for college entry and potentially avoid failure and remediation.

2. ***Consider making minor adjustments to improve the EAP. Build on the strengths of the EAP to further align expectations across the CSU, UC, and CCC for college readiness for entry-level coursework.***

Within this report reviewers have offered suggestions for minor adjustments to the EAP assessments in English and mathematics. There are also opportunities to build on the strengths of the EAP to further align expectations. For example, reviewers found strong alignment

between the EAP Essay and the California Content Standards in reading and writing. There are also similarities between the EAP Essay and the UC Analytic Writing Placement Examination (AWPE). This creates an opportunity for further comparisons of writing expectations for incoming freshmen across the CCC, CSU, and UC. A cross-segment group of faculty could conduct an analysis of direct writing assessment passages/prompts, rubrics, and anchor papers. The resulting common expectations for college-ready writing across all postsecondary segments should be made publicly available for K-12 students and educators statewide.

3. ***Devise a communication strategy for educators, students, and communities that clarifies the level of preparation that students must reach in order to demonstrate college readiness and the specific course in which they would be placed with successful completion of the EAP.***

Achieve found that across K-12 and postsecondary, communications and documentation about the EAP are extensive but not always consistent. The EAP has the potential for impacting more 11th graders throughout the state by signaling college readiness while students have time to address preparation in high school and potentially avoid remediation when entering postsecondary. It is important that the partners in this effort clarify the expectations students need to meet for success on the EAP in order to secure placement in credit-bearing coursework. In addition, students and educators may not understand what the first credit-bearing course is in English and mathematics on various UC, CSU, and CCC campuses and when and if there are exceptions. The expansion of the EAP to select community college campuses provides the opportunity to update public communications, websites, and other support materials with respect to the requirements and benefits of the EAP.

4. ***Encourage more students to take mathematics at and above the level of Algebra II.***

While the numbers of students taking the EAP has increased over the years, it is true in mathematics that this "early indicator" system is only available to those students who are enrolled in or have completed Algebra II or higher. Given that the state graduation requirements specify two units of mathematics including Algebra I, students may not have the incentive to go further than Algebra I—a level that falls far short of the required mathematics at California postsecondary campuses. While 79% of students participate in the English EAP, only 36% of students participate in the mathematics EAP. Both the English and mathematics EAP provide incentive and opportunity for students to learn more about their level of preparation and consider college as an option. State leaders should consider strategies for encouraging many more students to take this assessment or the potential for the EAP will not be fully realized.

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In conclusion, Achieve commends the leaders from the California State University System and the California Department of Education who have collaborated over the years in the creation of this unique approach to college-ready assessment. The EAP provides a national model - an "early indicator" of postsecondary preparation so that students can increase their knowledge

and skill while still in high school and avoid failure and remediation in their entry-level college experience. The expansion of the EAP beyond the CSU campuses provides the opportunity to engage other educators across the state in a systematic program to provide greater numbers of students with the opportunity to prepare for entry into credit-bearing coursework in postsecondary.

Introduction

The California Diploma Project (CDP) is focused on helping leaders in the state reach agreement on the use of a common assessment that would serve as an indicator of satisfactory progress toward college readiness. The proposed instrument is the augmented 11th grade California Standards Test (CST), which is the cornerstone of the Early Assessment Program (EAP). The California Diploma Project sought the assistance of Achieve in order to determine the degree to which the EAP tests in Algebra II, Summative Mathematics, and English provide an assessment of the level of college readiness required for entry-level coursework. For the purpose of this analysis of these EAP assessments, Achieve considered the California standards and additional expectations targeted by the CSU faculty as the benchmarks for college readiness – that is, as indicators of the level of preparation needed for entry into credit bearing coursework in English and mathematics on the CSU campuses.[1]

This analysis was preceded by a preliminary comparison of the *California Content Standards in English and Mathematics K-12* (adopted 1997) with the ADP Benchmarks in English and mathematics. There was a subsequent comparison of these standards with other frameworks of college readiness. A description of these prior analyses follows.

Preliminary Analysis

In October 2008, Achieve completed a comparison of the *California Content Standards in English and Mathematics K-12* (adopted 1997) with the ADP Benchmarks in English and mathematics. Achieve reviewers found that the California standards are well aligned with the ADP Benchmarks, with some minor exceptions. Given that a portion of the alignment came from California standards at the middle and upper elementary level, Achieve recommended that standards included in lower grades be fully reflected in later grades. No recommendations were made for revision of the California standards.

Phase I—Comparison of College Readiness Frameworks

A subsequent analysis of the California standards—Phase I of this study—was conducted in Spring 2009. This analysis included a crosswalk of the California Content Standards assessed on the EAP (as indicated on the assessment blueprints), the Statement of Competencies in academic literacy and mathematics from the Intersegmental Committee of the Academic Senates (ICAS), and the ADP Benchmarks. The review suggests that there is a shared set of college readiness expectations in the state, particularly in writing and the foundational skills found in Algebra I, Geometry, and Algebra II.

[1] See www.achieve.org/CaliforniaDiplomaProject for supplemental materials that crosswalk the California Content Standards for K-12 with ELM specifications (in mathematics), the ICAS competencies, and the American Diploma Project Benchmarks.

Phase II—Analysis of the EAP Assessments in Algebra II, Summative Mathematics, and English

This analysis focuses on the degree to which the EAP tests in Algebra II, Summative Mathematics, and English provide an assessment of the level of college readiness required for entry-level coursework. A detailed description of the standards targeted for assessment by the EAP for Algebra II (Appendix A), Summative Mathematics (Appendix B), and English (Appendix C). As noted, each of the EAP assessments includes items selected from the CST by the CSU faculty, however, this analysis should not be interpreted as an evaluation of the entire CST as this is neither the intent nor an accurate picture of the assessments examined.

This analysis focuses on the following questions related to entry-level college readiness (as defined by the standards targeted for assessment by CSU faculty):

- How do the specific areas of students' knowledge and skill that are assessed in English and mathematics align with entry-level college readiness?
- To what degree is students' knowledge of content as well as performance assessed and aligned with entry-level college readiness?
- At what level of challenge and cognitive demand do the items assess student knowledge and performance?
- Is the level of reading comparable to that required at an entry-level college readiness level?
- What are the characteristics of the direct writing assessment that are indicative of authentic, college-level writing?
- Are there areas for improvement?

The Early Assessment Program (EAP)

The EAP serves as a college readiness signal for students while still in high school, enabling them to better prepare for college in their senior year. The test is voluntary and is only available to students in the 11th grade. The EAP is a combination of select items from the California Standards Test (CST)—in Algebra II, Summative High School Mathematics (Summative Math), and Grade 11 English-Language Arts (ELA)—and a set of 15 items developed by faculty at CSU. In English, the EAP also includes a direct writing assessment.

The CSTs measure student progress toward achieving California's state-adopted academic content standards, which describe what students should know and be able to do in each grade and subject tested. Students in grades two through eleven take a set of CSTs in various subject areas. Eleventh grade students who choose to take the EAP will receive a college readiness score as part of their STAR report, an annual progress report on CST performance sent to students and parents.

Mathematics

The 15 augmented items are added to the Algebra II and Summative Mathematics CSTs. Note that only students who are enrolled in or have completed Algebra II may take these CSTs, and

thus the EAP. The Algebra II CST assesses content from the California Content Standards in the Algebra II and Probability and Statistics standards. The Summative Mathematics is intended to assess content across Algebra I, Geometry, Algebra II, and Probability and Statistics standards from the California Content Standards. While the full CSTs consist of 65 items, only a subset of these items are included as part of a student's college readiness score. The subset of items is selected by a Test Committee based on content addressed on the Entry Level Mathematics (ELM)[2]. For example, the CST items that address probability and statistics are not counted towards a student's EAP score. The 15 augmentation items are developed for alignment to the ELM specifications in Algebra and Geometry and are the same item set for both CSTs.

Based on performance on the subset of CST items and 13 augmentation items[3], students receive one of three college readiness scores in mathematics.

1. Ready for College: Students have met the CSU expectations in mathematics for incoming freshmen and are ready for the first-year, credit-bearing mathematics course. Students are exempt from placement tests at any CSU campus.
2. Ready for College—Conditional: Students are on track to be prepared by the time they graduate high school, given they take a mathematics course in their senior year. Students who enroll and earn a C or better in a mathematics course—with an Algebra II prerequisite—in their senior year will be exempt from placement tests at any CSU campus.
3. Did Not Demonstrate College Readiness: Students will need to take the ELM placement exam upon enrollment at a CSU campus.

English

The 15 augmentation items are added to the Grade 11 English-Language Arts CST. All students are required to take the assessment. Like mathematics, a faculty Test Committee selects a subset of the 75 CST items—based on English Placement Test (EPT) [4] content—to include as part of a student's EAP score. The numbers of items range from 33-55 and are ideally around 42. The augmented items are released items from CSU's EPT and focus on demonstration of critical reading and writing skills.

To mirror the placement requirements of incoming freshmen, CSU faculty added to the EAP a 45-minute direct writing assessment that asks students to read a short passage and essay prompt, to think and analyze, and to write an essay in response to the prompt.

Upon completing all three parts of the EAP (CST, augmented items, and essay), students are notified of their college readiness score.

[2] The ELM is the placement instrument used by CSU to place incoming freshmen into first-year mathematics courses.
[3] Two of the 15 augmented items are field test items.
[4] The EPT is the placement instrument used by CSU to place incoming freshmen into first-year English courses.

1. Ready for College: Students have met the CSU EPT requirements and are ready for credit-bearing English courses upon enrollment at CSU. Students are exempt from taking the EPT for placement.
2. Did Not Demonstrate College Readiness: Students are not yet ready to succeed in college level English courses. Students will be required to take the EPT for placement.

Representative forms provided to Achieve

One representative form for each assessment—English Language Arts, Algebra II, and Summative High School Mathematics—was reviewed. Due to state policy restrictions, Achieve was unable to review full, intact forms of the CST assessments. This analysis therefore, should not be interpreted as a review of the CST but rather as a review of the EAP assessments that are comprised of a subset of CST items and augmentation. The CST items as well as the augmented items and writing samples were submitted to Achieve by the Educational Testing Service (ETS), the organization responsible for developing the CST, as representative of the items used by CSU to arrive at a student's EAP college readiness score. A description of each of the EAP Assessments follows.

Algebra II EAP

The Algebra II EAP representative test form included CST Algebra II items that were used as part of a previous EAP test, but were not all used on the same test form. The CST items provided are those that have been released to the public. Using released test items limited the items available for this study. According to ETS, the items are statistically representative of the test and the test blueprint, but these items probably would not have been used for the same test administration. The Algebra II EAP augmentation items were used as part of the 2007 EAP test administration. The EAP math augmentation items are the same for both Algebra II and Summative Mathematics.

Summative Mathematics EAP

The Summative Mathematics EAP items were selected from the pool of released mathematics test questions that met the specifications for EAP but may not have been specifically selected for use in a CST or Summative Mathematics EAP test. According to ETS, this ad hoc form met the statistical requirements of both the CST and EAP, and it matched the test blueprint. The EAP augmentation items were used as part of the 2007 EAP test administration. The EAP mathematics augmentation items are the same for both Algebra II and Summative Mathematics.

English EAP

The English EAP representative test form included CST ELA items that were used as part of a previous EAP test, but were not all used on the same test form. The CST items provided for Achieve's purposes have been released to the public or were associated with passages that were released. Using released test items limited the items available for this study. According to ETS, the items are statistically representative of the test and the test blueprint, but these

items probably would not have all been used for the same test administration. The English EAP augmentation items and essay topics were used as part of the 2007 EAP test administration.

Methodology—Achieve Assessment to Standards (ATS) Alignment Protocol

To conduct the analysis, Achieve convened a group of content experts in each subject area—English and mathematics. The assessment panels followed the Achieve Assessment to Standards (ATS) Protocol, developed by Achieve with assistance from psychometricians and content experts. Panel members independently reviewed the CST and augmented items and then, through detailed discussion of each item, reconciled any differences in judgments. The protocol considers several criteria in determining the degree of alignment between an assessment and standards.

- ***Confirmation of the test blueprint***

Achieve analyzed several documents including the blueprint and item card in order to validate the mapping of an item to a standard. The assessment blueprint shows the intended match between a test item and the standards. Reviewers judge whether or not the item is appropriately matched to a standard. Comparisons of the state's blueprint for the CST with the representative forms of the EAP analyzed by Achieve for this report provide a perspective on the standards targeted for assessment by the EAP for Algebra II (Appendix A), Summative Mathematics (Appendix B), and English (Appendix C).

- ***Content centrality***

This criterion analyzes the match between the content that is targeted by a test question and the content described in a standard. Reviewers assign a content centrality score to each item that reflects the degree or quality of alignment between the content described in a standard to that addressed by the item as follows:

- 2 - clearly consistent
- 1a - not specific enough (standard or objective is too general to be assured of item's clear consistency)
- 1b - somewhat consistent (item assesses only part of a compound standard. In mathematics, the item assesses the less central part of the standard)
- 0 - inconsistent

A score of 2 is awarded when the standard is specific and the item clearly measures the content spelled out in the standard. A score of 1a indicates that the standard to which the item was mapped is not effectively communicating the content to be addressed. A high number of 1a scores, therefore, signal concerns about the clarity of the standard. Items that receive 1b scores are examined as a set when discussing balance of the assessment. Any item that receives a 0 is reviewed against the other standards for potential matches. If a better match is found, the item is then scored for its alignment to the remapped standard.

- ***Performance centrality***

This criterion focuses on the degree of the match between the type of performance required by a test item and the performance required by the related standard. Each item elicits a type of cognitive performance (e.g., the item requires that the test taker "select," "identify," "represent," or "analyze"). Reviewers assign a performance centrality score for each item based on whether there is a clear consistency between the type(s) of performance demanded by the item and the type(s) of performance described in the standard as follows:

- 2 - clearly consistent
- 1a - somewhat consistent/not specific enough (standard or objective is too general to be assured of item's clear consistency/strong alignment)
- 1b - somewhat consistent (standard or objective uses more than one verb, but the item matches the less cognitively demanding of the verbs)
- 0 - inconsistent

- ***Source of challenge***

This judgment attempts to uncover whether individual test items are "fair." The question asked about an item's source of challenge is whether or not choosing the correct answer is a probable indication that the skill being targeted is actually the skill being assessed.

Items with appropriate sources of challenge help ensure two things: (a) that a student who does well on the item probably has a good grasp of the content targeted, and (b) that a student who does poorly on the item probably has a weak grasp on the content targeted. Said another way, analyzing source of challenge helps to identify those questions where a student may get the *right* answer for the wrong reason (false positive), or the *wrong* answer but in fact has the knowledge to answer the item correctly (false negative). In an item with appropriate sources of challenge, the greatest challenges in the item lie in the content and type of performance targeted in the relevant objective(s) as opposed, for example, to challenges of background knowledge or interpretation of the item's context.

Reviewers rate each item for source of challenge as follows:

- 1 - appropriate source(s) of challenge
- 0 - inappropriate source(s) of challenge

- ***Level of Cognitive Demand***

Judgments about level of cognitive demand focus on the type and level of thinking and reasoning required by students on individual items.

In mathematics, the protocol differentiates between four levels of cognitive demand, ranging from the simple recall of information to complex reasoning and thinking. Complexity of the mathematics determines the cognitive level, not difficulty for students.

- Level 1 (recall) – item requires the recall of information such as a fact, definition, term, or simple procedure. Level 1 items are usually rote and procedural in nature, such as performing an algorithm or applying a formula or property.
- Level 2 (skill/concept) – item calls for the engagement of some mental processing beyond a habitual response, with students required to make some decisions as to how to approach a problem or activity.
- Level 3 (strategic thinking) – item requires students to reason, explain their thinking, plan, conjecture or use evidence. Level 3 items are non-routine or strategic in nature. Students must establish meaning.
- Level 4 (extended/abstract thinking) – item requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time.

In English, the levels represent a hierarchy based on Bloom's Taxonomy that describes levels of sophistication and complexity of thinking. The sophistication will depend on the degree to which simple knowledge and skills have to be recalled or drawn upon, the amount of cognitive processing required, the degree to which the task requires going beyond the text, and the need to extend or produce novel findings.

- Level 1 (recall or locate) – item requires little beyond simple recall or identification such as identify figurative language or basic facts in a reading passage.
- Level 2 (infer or integrate) – item demands a level of inference involving some comprehension and subsequent processing of texts, such as using context clues to identify unfamiliar words, predicting logical outcomes, or deciding appropriate verb tense by considering the meaning of a sentence.
- Level 3 (analyze or apply) – item requires analysis of the text and an understanding of how a passage works. Level 3 items include determining author's purpose, understanding the use of symbols, or accomplishing a rhetorical purpose.
- Level 4 (critique or evaluate) – item requires evaluation of material and are often open-ended.

In English, reviewers also evaluate the level of demand of the reading passage. Here, reviewers analyze various elements of the passage including vocabulary, sentence structure, and the skill in literary analysis required.

- ***Balance***

Balance compares the emphasis of content offered by an item set to the emphasis of content described by the standards. In addition to evaluating alignment, reviewers also judge whether the set of items emphasizes the more important content at the grade level.

- ***Level of Challenge***

This criterion is applied to a *set* of items, and reviewers consider whether the set of items mapped to a standard represent an appropriate range of challenge.

The criteria and process that have been developed unpack different factors that contribute to the quality of the alignment between assessments and standards, thereby providing a level of information about assessments typically unavailable to states and organizations.[5] It is important to recognize that the process relies on the *clinical judgments* of experienced content and assessment experts. The criteria and the training of reviewers are rigorous, and seasoned judgment is a major aspect of the review.

Those who are charged with the actual development/selection of the EAP items have been provided with a detailed technical review of the findings of this analysis. A summary of major findings by criteria for the Mathematics (Algebra II and Summative Mathematics) EAP Assessments follows.

[5] It is assumed that any state or organization evaluating a particular exam will run its own bias review, drawing upon the work of a representative and well-qualified panel of experts, so this aspect of an assessment is not a part of this protocol.

Findings Mathematics

Achieve found:

I. The Algebra II and the Summative Mathematics EAP assessments address essential college-ready mathematics identified by the CSU faculty for entry into credit bearing coursework.
II. The augmentation items—which are the same for the Algebra II and the Summative Mathematics EAP assessments—expect students to make connections across mathematics and raise the level of cognitive demand for the collective set of EAP test items.

Achieve recommends that the EAP Test Committee consider:

1. Including/developing a number of more cognitively demanding selected-response items in order to assess thinking at the strategic level.
2. Exploring the addition of a limited number of constructed response items.
3. Screening items for complicated or lengthy arithmetic that presents an inappropriate source of challenge for students without calculators.
4. Selecting items that would balance coverage to include other important college-ready content that is in the CST blueprint.

This section presents findings relative to the entire EAP assessment (CST item subset and augmentation items combined) in Algebra II followed by findings relative to the Summative Mathematics EAP. For each of these EAP assessments, Achieve reviewers first considered the CST subset items, then the set of augmentation items and finally the combined set of items—presented here—that are used to generate a student's EAP college readiness score. Given that the partners in the California Diploma Project are considering the expansion of the EAP, we conclude with recommendations for making this college ready assessment even better.

Algebra II EAP

What Mathematics Content Does the Algebra II EAP Cover?

The Algebra II EAP (CST item subset and augmentation items combined) covers a range of the California Content Standards. Although the CST item subset covers only Algebra II content, the augmentation items expand coverage to include Algebra I and Geometry. In Algebra II, students are asked to solve systems of linear equations, understand and use exponential functions, simplify rational expressions with monomial denominators, and solve quadratic equations. In Algebra I, students are asked about rules of exponents. In Geometry, students are asked to compute areas of geometric figures. See Appendix A for the full listing of standards that make up the content and performance expectations and blueprint for the Algebra II EAP.

The following tables and narrative summarize ratings for content centrality, performance centrality, source of challenge, and level of cognitive demand when the 45 CST Algebra II items and the 13 EAP augmentation items are combined. Such a tally offers perspective on the characteristics of the item pool used to determine student EAP scores. In addition, by identifying how the 13 items in the EAP augmentation set align not with the ELM Specifications for EAP but with the California Content Standards—particularly the Algebra II standards—it is possible to get a sense of the impact the augmentation set has on the balance and level of challenge of the collection of items used to generate student EAP scores. All tables are calculated using the revised mappings proposed by Achieve.[6]

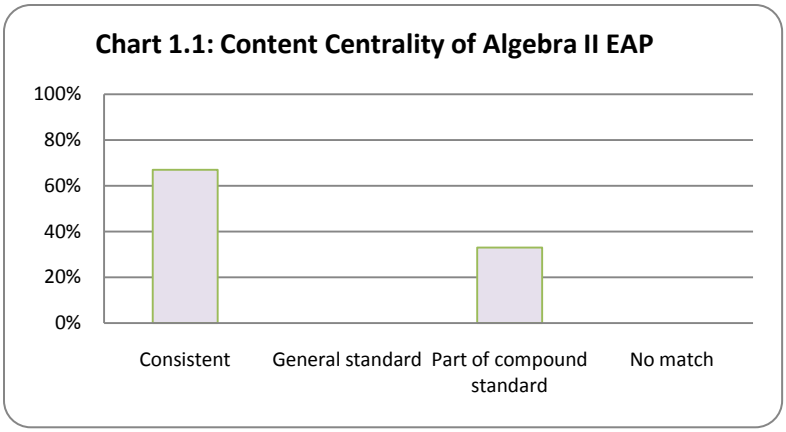
Content Centrality

In terms of content centrality, 67% of the items that were mapped to the California Algebra II standards and the ELM specifications—as defined by the test blueprint—were determined to consistently match the intent of the standard. The other 33% of the items target partial elements of the standards to which they were mapped (Table 1.1). Those with an interest in the EAP test results can be assured that 100% of the EAP items target quite clearly the content described in the standards.

Table 1.1: Content Centrality of Algebra II EAP

California Content Standards: Algebra II and ELM Specifications	*Number of items*	*Content Centrality*			
		2 ***Consistent***	***1a*** ***General standard***	***1b*** ***Part of compound standard***	***0*** ***No match***
California Content Standards: Algebra II	45	28 (62%)	0	17 (38%)	0
ELM Specifications for EAP	13	11 (85%)	0	2 (15%)	0
Totals	**58**	**39**	**0**	**19**	**0**
Percent	**100%**	**67%**	**0%**	**33%**	**0%**

[6] Reviewers remapped three items to other standards in the Algebra II blueprint and ELM Specifications. In addition, two of the 58 EAP items exhibited inappropriate sources of challenge.



Note in Table 1.1 that a third of the items (33%) were determined to partially address the standard to which they were mapped. While still a strong rating, these items need to be viewed as a set of items that together address the total content of the standards. Table 1.2 highlights a few examples of these items and provides details on the aspects of content that are—and that are not—assessed.

Table 1.2: Examples of Algebra II EAP Items with 1b Content Rating

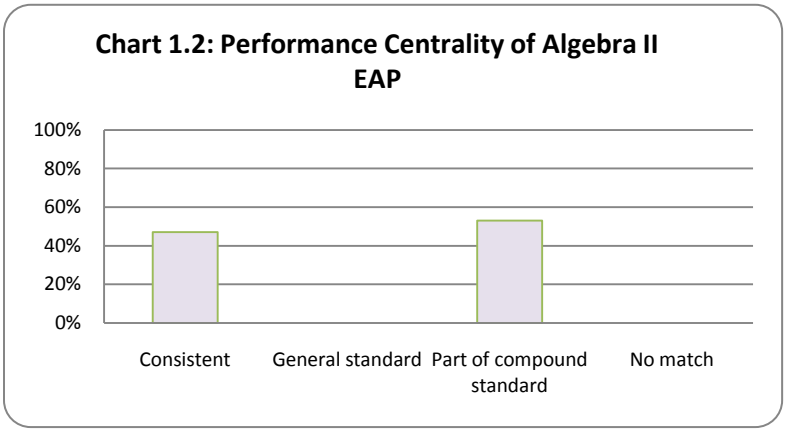
Content Standard	Items Aligned to this Standard	Items Rated as 1b	Assessed Content	Comments
2A2.0* Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.	5	2	The two items that were rated 1b both assess systems of equations.	Both systems of linear equations and inequalities are assessed across the five items. All items involve two variables. Solution strategies are neither specified nor easily assessable, given the multiple-choice format of the test. Two items involve graphs.
2A4.0* Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.	3	3	Two items assess the difference of squares, and one item assesses perfect square trinomials.	No items assess the sum and difference of two cubes.
A2EAP4.4 Find real solutions to simple equations (including quadratic and rational), with both numerical and literal coefficients.	3	2	The two items that were rated 1b both assess numerical coefficients only.	Across the three items, both numerical and literal coefficients are included.

Performance Centrality

In terms of performance centrality, 47% of the items that were mapped to the California Algebra II standards and the ELM Specifications—as defined by the test blueprint —were determined to consistently match the intent of the standard (Table 1.3). The other 53% of the items target partial elements of the standards to which they were mapped. Those with an interest in the EAP test results could be assured that 100% of the EAP items target performances described in the standards.

Table 1.3: Performance Centrality of Algebra II EAP

California Content Standards: Algebra II and ELM Specifications	*Number of items*	*Performance Centrality*			
		2 Consistent	***1a General standard***	***1b Part of compound standard***	***0 No match***
California Content Standards: Algebra II	45	16 (36%)	0	29 (64%)	0
ELM Specifications for EAP	13	11 (85%)	0	2 (15%)	0
Totals	**58**	**27**	**0**	**31**	**0**
Percent	**100%**	**47%**	**0%**	**53%**	**0%**



Note in Table 1.3 that over half of the reviewed items (53%) were determined to partially address the standard to which they were mapped. While still a strong rating, these items must be viewed as a set to determine the balance of the performance expectations. Table 1.4 provides a few examples.

Table 1.4: Examples of Algebra II EAP Items with 1b Performance Rating

Content Standard	Items Aligned to this Standard	Items Rated as 1b	Assessed Performance	Comments
2A6.0* Students add, subtract, multiply, and divide complex numbers	3	3	Two items assess multiplication of complex numbers, while the other assesses subtraction.	No division of complex numbers is assessed. Addition is embedded within the subtraction problem.
2A22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.	2	2	One item assesses finding the general term, while the other addresses finding the sum.	Together, both items adequately assess the performance expectations of this standard.
A2EAP4.1 Evaluate & interpret algebraic expressions (including function notation, rational exponents, and radicals).	2	2	One item calls for evaluation while the other item calls for interpretation.	Across the two items both performances are assessed.

Level of Cognitive Demand

With 59% of the items categorized by reviewers as level 1, these Algebra II EAP items tend to be of a procedural nature (Table 1.5). In many cases, an item involving more rigorous mathematical topics (such as multiple representations) is asked at a procedural level, thereby lowering its cognitive demand. The other 41% of the Algebra II EAP items go beyond procedural mathematics and require students to think, plan, and execute. Some of these items are presented in a context that expects students to make meaning of the mathematics and then mathematically model the situation.

It is worthy of note that with three level 1 ratings and 10 level 2 ratings, the set of 13 augmentation items raises the overall cognitive demand of the Algebra II EAP. The number of items rated level 2 increases substantially, from 31% to 41%. The augmentation items call on students to make decisions about how to approach a problem and to display conceptual understandings. These items generally mirror the levels of performance represented in the ELM specifications for the Algebra II EAP.

Table 1.5: Cognitive Demand of Algebra II EAP

California Content Standards: Algebra II and ELM Specifications	*Number of items*	*Cognitive Demand*			
		1 Procedural	*2 Skill/ concept*	*3 Strategic*	*4 Abstract*
California Content Standards: Algebra II	45	31 (69%)	14 (31%)	0	0
ELM Specifications for EAP	13	3 (23%)	10 (77%)	0	0
Totals	**58**	**34**	**24**	**0**	**0**
Percent	**100%**	**59%**	**41%**	**0%**	**0%**

Chart 1.3: Cognitive Demand of Algebra II EAP

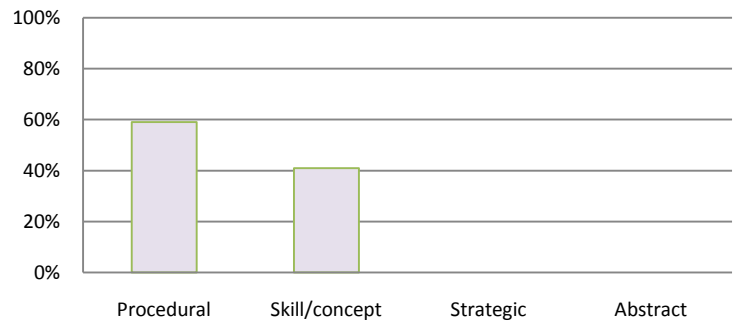


Table 1.6 highlights the differences between a level 1 item and a level 2 item.

Table 1.6: Example of Cognitive Demand Comparison of Algebra II EAP Items

Item	Content Standard	Cognitive Demand
Which point lies in the solution set for the system $\begin{cases} 2y-x \geq -6 \\ 2y-3x < -6 \end{cases}$?	2A2.0* Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with matrices.	1
A restaurant manager bought 20 packages of bagels. Some packages contained 6 bagels each, and the rest contained 12 bagels each. There were 168 bagels in all. How many packages of 12 bagels did the manager buy?		2

Reviewers noted that the augmentation items have been written to make connections across and/within the domains of algebra and geometry, and it is the connections that these items require students to make that contribute to their level of challenge as an item set. Table 1.7 gives brief descriptors of each item, highlighting instances where multiple mathematical concepts are involved, requiring students to make connections.

Table 1.7: Characteristics of Augmentation Items

Item	Comments on Item Characteristics Contributing to Level of Challenge
1	This item requires students to evaluate a function involving the application of rules of exponents.
2	This item requires students to display understanding of the rules of radicands.
3	This item involves use of a literal coefficient in a quadratic equation and calls on students to have an understanding of the quadratic formula and the role of the discriminant in determining number of solutions.
4	This is the only item that involves solving an equation with rational exponents.
5	This is one of the few items cast in a real world context causing students to write the equations that model the situation and then solve them and make sense of the solution in the original context.
6	This item involves both linear and quadratic equations on the coordinate plane and calls for students to understand the geometry of a circle. It has multiple solution paths.
7	A procedural item was enhanced by involving reasoning about what values of the denominator of a fraction will result in a value greater than 0 before solving an absolute value equation in the denominator.
8	This item requires students to make connections between systems of linear equations, coordinate geometry, and area.
9	This item calls on students to understand the coordinate graphs of functions and the relationship between the graph of a function and the graph of the reciprocal of that function.
10	This item involves area, perimeter, and the Pythagorean Theorem (or knowledge of properties of special right triangles).
11	Using visualization this item is easily solved without tedious computation involving the area of a circle. A less able or perceptive student could solve the problem by performing multiple area calculations.
12	This item extends the concept of similarity to solid figures involving scaled volumes.
13	This item requires students to apply both geometry and algebra to the solution of a problem. Given area, students must find the height by solving a quadratic equation, eliminating an extraneous solution and then making sense of the answer in the original context.

Balance

The Algebra II EAP is intended to address content primarily from the Algebra II strand (79%). However since the augmented items make connections across several standards, the EAP also covers Algebra I (7%) and Geometry (14%).

Level of Challenge

The entire Algebra II CST blueprint provides the full range of content from which the EAP Test Committee may select items. The standards on the Algebra II CST blueprint provide opportunities for the inclusion of items that can be both appropriate to this college-ready assessment while sampling various levels of challenge from procedural to complex. See Appendix A.

Summative Mathematics EAP

What Mathematics Content Does the Summative Mathematics EAP Cover?

The Summative Mathematics EAP covers a range of the California Content Standards in Algebra I, Algebra II, and Geometry. In Algebra I, the Summative Mathematics EAP addresses rules of exponents, rate problems, and polynomials. In Geometry, students are asked to compute various measurements of geometric figures and use the Pythagorean Theorem. The Algebra II items address solving quadratic and linear equations and using exponential functions. See Appendix B for the full listing of standards that make up the content and performance expectations and blueprint for the Summative Mathematics EAP.

The following tables and narrative summarize ratings for content centrality, performance centrality, source of challenge, and level of cognitive demand when the 49 CST Summative Mathematics items and the 13 EAP augmentation items are combined. Such a tally offers perspective on the characteristics of the item pool used to determine student EAP scores. In addition, by identifying how the 13 items in the EAP augmentation set align not with the ELM specifications for EAP but with the California Content Standards—particularly those included on the Summative Mathematics assessment—it is possible to get a sense of the impact the augmentation set has on the balance and level of challenge of the collection of items used to generate student EAP scores. All tables are calculated using the revised mappings proposed by Achieve.[7]

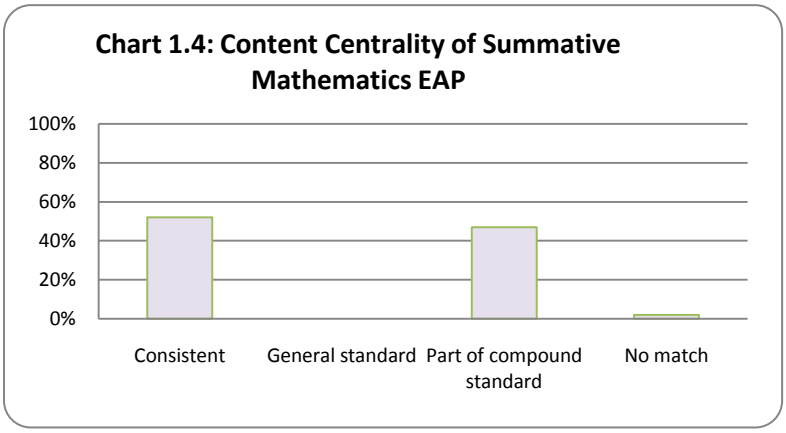
Content Centrality

In terms of content centrality, 51% of the items that were mapped to the California mathematics standards and the ELM Specifications—as defined by the test blueprint—were determined to consistently match the intent of the standard. Another 47% target partial elements of the standards to which they were mapped. Those with an interest in EAP test results can be assured that 98% of the item subset target quite clearly the content described in the standards.

[7] Reviewers remapped two items to other standards. In addition, one of the 62 EAP items exhibited inappropriate source of challenge.

Table 1.8: Content Centrality of Summative Mathematics EAP

California Content Standards: Summative Mathematics and ELM Specifications	*Number of items*	*Content Centrality*			
		2 Consistent	***1a General standard***	***1b Part of compound standard***	***0 No match***
California Content Standards: Summative Mathematics (Algebra I, Geometry, Algebra II)	49	21 (43%)	0	27 (55%)	1 (2%)
ELM Specifications for EAP	13	11 (85%)	0	2 (15%)	0
Totals	**62**	**32**	**0**	**29**	**1**
Percent	**100%**	**51%**	**0%**	**47%**	**2%**



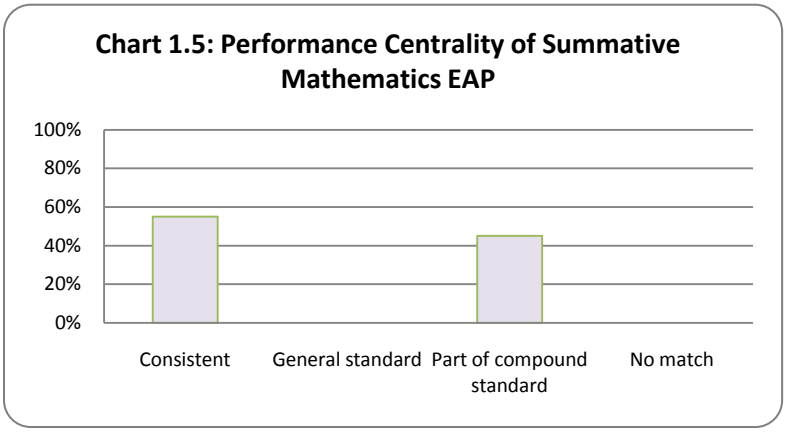
Performance Centrality

In terms of performance centrality, 55% of the items that were mapped to the California mathematics standards and the ELM Specifications—as defined by the test blueprint —were determined to consistently match the intent of the standards (Table 1.9). The other 45% of the items target partial elements of the standards to which they were mapped. Those with an interest in EAP test results could be assured that 100% of the EAP items target performances described in the standards.

Table 1.9: Performance Centrality of Summative Mathematics EAP

California Content Standards: Summative Mathematics and ELM Specifications	*Number of items*	*Performance Centrality*			
		2 Consistent	***1a General standard***	***1b Part of compound standard***	***0 No match***
California Content Standards: Summative Mathematics (Algebra I, Geometry, Algebra II)	49	23 (47%)	0	26 (53%)	0
ELM Specifications for EAP	13	11 (85%)	0	2 (15%)	0
Totals	**62**[a]	**34**	**0**	**28**	**0**
Percent	**100%**	**55%**	**0%**	**45%**	**0%**

[a] *One of the 62 EAP items was found to have an inappropriate source of challenge.*



Level of Cognitive Demand

With 55% of the items categorized by reviewers as level 1, a majority of the Summative Mathematics EAP items tend to be of a procedural nature (Table 1.10). Similar to items in the Algebra II EAP, in many cases, the Summative Mathematics EAP items involve more rigorous mathematical topics (such as exponential functions) but are asked at a procedural level, thereby lowering the cognitive demand. The other 45% of the Summative Mathematics EAP items go beyond procedural mathematics and require students to think, plan, and execute. Some of these items are presented in a context that expects students to make meaning of the mathematics and then mathematically model the situation.

Table 1.10: Cognitive Demand of Summative Mathematics EAP Items

California Content Standards: Summative Mathematics and ELM Specifications	*Number of items*	*Cognitive Demand*			
		1 Procedural	***2 Skill/ concept***	***3 Strategic***	***4 Abstract***
California Content Standards: Summative Mathematics (Algebra I, Geometry, Algebra II)	49	31 (63%)	18 (37%)	0	0
ELM Specifications for EAP	13	3 (23%)	10 (77%)	0	0
Totals	**62**	**34**	**28**	**0**	**0**
Percent	**100%**	**55%**	**45%**	**0%**	**0%**

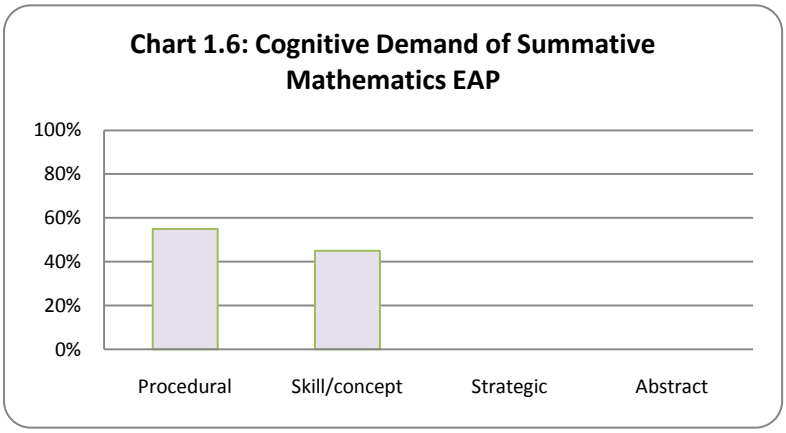


Table 1.11 highlights the differences between a level 1 item and a level 2 item.

Table 1.11: Example of Cognitive Demand Comparison of Summative Mathematics EAP Items

Item	Content Standard	Cognitive Demand
Which of the following expressions is equal to $(x + 2) + (x – 2)(2x + 1)$?	1A10.0* Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.	1
Beth is two years older than Julio. Gerald is twice as old as Beth. Debra is twice as old as Gerald. The sum of their ages is 38. How old is Beth?	1A5.0* Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	2

As with the Algebra II EAP, Achieve reviewers found that the addition of the 13 augmentation items raises the overall cognitive demand of the Summative Mathematics EAP. The number of items rated level 2 increases substantially, from 37% to 45%. Many of the augmentation items require students to make connections both within and across the algebra and geometry domains, which adds to the richness, and hence cognitive demand, of the items. See Table 1.7 in the Algebra II EAP section for an in-depth description of the augmentation items.

Balance

The Summative Mathematics EAP addresses content across three sets of standards—Algebra I, Geometry, and Algebra II. Items are balanced across these standards with 40% covering Algebra II content, 30% addressing Algebra I content, and the remaining 30% covering Geometry content.

Level of Challenge

The entire Summative Mathematics CST blueprint provides the full range of content from which the EAP Test Committee may select items. The standards on the Summative Mathematics EAP blueprint provide opportunities for the inclusion of items that can be both appropriate to this college-ready assessment while sampling various levels of challenge from procedural to complex. See Appendix B.

Concluding Recommendations—Mathematics

Achieve completed this analysis of the Algebra II and the Summative Mathematics EAP assessments at the request of the partners who lead this effort for the CSU and the CDE. Given that the partners in the California Diploma Project are considering the expansion of the EAP, recommendations have been offered for their consideration.

1. ***The EAP Test Committee should consider including/developing a select number of more cognitively demanding selected-response items in order to assess thinking at the strategic level.***

   A particular strength of the augmentation item set is that many items require students to make connections across and/or within mathematical domains. These items tend to be more interesting and mathematically richer and can be used as models for developing future items for both the Algebra II and Summative Mathematics EAP. The inclusion/development of additional selected-response items that resemble these, plus others that assess higher levels of cognitive demand, would help ensure that these items assess the kinds of higher order skills students need to be prepared for college. Finally, the inclusion of application problems from the social sciences or sciences would also enhance this pool of items, enabling students to demonstrate their abilities to use mathematics to model relevant situations with functions.

2. ***Consider exploring the addition of a limited number of constructed response items.***

The addition of constructed-response, or open-ended, items—even a limited number of them—would be a vehicle for increasing the cognitive demand level of this collective set of items. While it is true that such items can be expensive, given costs of scoring, they are critical in assessing skills that are difficult, to assess through multiple-choice items. Such items allow teachers to see first-hand how students approach solving problems, whether they can create graphs on their own, whether they can create conjectures or justifications on their own, and whether they can explain their reasoning.

3. ***Screen items for complicated or lengthy arithmetic that present an inappropriate source of challenge for students without calculators.***

   Some items involve complicated or lengthy arithmetic, and if such items are to continue being a part of this assessment, consideration should be given to allowing calculator use. If calculator use on all or part of the test is not an option, then items need to be screened to ensure that the arithmetic does not present an inappropriate source of challenge for students.

4. ***Consider the inclusion/selection of items that would balance coverage to include other important college-ready content that is in the CST blueprint.***

   Specific feedback related to content emphasis and to particular standards is as follows:
   - inequalities, volume of geometric figures, and laws of fractional exponents—aspects of assessed standards that are not adequately represented in the subset of 49 Summative Mathematics CST items examined.
   - properties of logarithms and the inverse relationship between exponents and logarithms. (from 2A11.1* and 2A14.0, two Algebra II standards included in both CST blueprints but not identified for inclusion in the EAP. Given the demands of College Algebra, some familiarity with these concepts is important for college readiness.)
   - transfer between/among representations as a means of gauging students' conceptual understanding.
   - probability and statistics. (These concepts are addressed in both the CST blueprints through 2A19.0* and three Probability and Statistics standards (1.0, 2.0, and 7.0) but not identified for inclusion in the EAP. With these domains of mathematics becoming increasingly important for students, particularly in non-STEM majors such as the social sciences, it makes sense to include some such items in the EAP components.)

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Appendix A: Standards Assessed on the Algebra II Early Assessment Program

as defined by the representative form provided to Achieve for this analysis

This Appendix indicates the California Content Standards included in the entire Algebra II CST blueprint. Also included are the number of items aligned to the California Content Standards in the CST Subset, the augmentation item set, and the total reviewed Algebra II EAP.

California Content Standards		CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
Algebra II					
1.0*	Students solve equations and inequalities involving absolute value.	1	1	1	2
2.0*	Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.	5	5	1	6
3.0*	Students are adept at operations on polynomials, including long division.	4	4	0	4
4.0*	Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.	3	3	0	3
5.0*	Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.	2	2	0	2
6.0*	Students add, subtract, multiply, and divide complex numbers.	3	3	0	3
7.0*	Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.	6	6	0	6
8.0*	Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.	4	4	3	7
9.0*	Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x-b)^2 + c$.	2	1	1	2

California Content Standards	CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
10.0* Students graph quadratic functions and determine the maxima, minima, and zeros of the function.	3 (2/3)[e]	1	0	1
Standard Set 11.0* Students prove simple laws of logarithms.				
11.1* Students understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.	3	0	0	0
11.2* Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.	3	0	0	0
12.0* Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.	3	3	2	5
13.0 Students use the definition of logarithms to translate between logarithms in any base.	1	0	0	0
14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.	2	0	0	0
15.0* Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.	4	3	0	3
16.0 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.	1/3[e]	0	0	0
17.0 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.	1	2	1	3
18.0* Students use fundamental counting principles to compute combinations and permutations.	2	2	0	2

California Content Standards		CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
19.0*	Students use combinations and permutations to compute probabilities.	2	0	0	0
20.0*	Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.	2	2	0	2
21.0	Students apply the method of mathematical induction to prove general statements about the positive integers.	1/3[e]	0	0	0
22.0	Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.	2	2	0	2
23.0*	Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.	NA	0	0	0
24.0	Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.	1/3[e]	1	2	3
25.0	Students use properties from number systems to justify steps in combining and simplifying functions.	1/3[e]	0	0	0
Probability and Statistics					
1.0	Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.	1	0	0	0
2.0	Students know the definition of *conditional probabilit*y and use it to solve for probabilities in finite sample spaces.	2	0	0	0
7.0	Students compute the variance and the standard deviation of a distribution of data.	2	0	0	0
Algebra I *(the standards below were determined by Achieve reviewers to map to the augmentation items)*					
2.0	Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.	0	0	2	2
3.0	Students solve equations and inequalities involving absolute values.	0	0	1	1
15.0	Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems	0	0	1	1
22.0	Students use the quadratic formula or	0	0	1	1

California Content Standards		CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
	factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points				
Geometry *(the standards below were determined by Achieve reviewers to map to the augmentation items)*					
8.0	Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figure.	0	0	4	4
10.0	Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids	0	0	3	3
11.0	Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.	0	0	1	1
17.0	Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.	0	0	1	1
22.0	Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.	0	0	1	1

**Defined by the California Department of Education as a Key Standard. Key standards must make up a minimum of 70% of the CST.*

[a] *CSU EAP Test Committee is able to select from the full range of standards assessed on the CST*

[b] *Blueprint reflects items re-mapped during the analysis*

[c] *Achieve reviewers mapped the augmentation items to the California Content Standards. Some items address several standards.*

[d] *The final column includes the CST subset and the augmentation items.*

[e] *Fractional values indicate rotated standards (e.g., 1/2 = rotated every two years)*

Appendix B: Standards Assessed on the Summative Mathematics Early Assessment Program

as defined by the representative form provided to Achieve for this analysis

This Appendix indicates the California Content Standards included in the entire Summative Mathematics CST blueprint. Also included are the number of items aligned to the California Content Standards in the CST Subset, the augmentation item set, and the total reviewed Summative Mathematics EAP.

California Content Standards		CDE CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
Algebra I					
2.0	Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.	0	0	2	2
3.0	Students solve equations and inequalities involving absolute values.	0	0	1	1
4.0*	Students simplify expressions prior to solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.	1	1	0	1
5.0*	Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	3	3	0	3
6.0*	Students graph a linear equation and compute the *x*- and *y*- intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).	2	2	0	2
7.0*	Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations using the point-slope formula.	1	1	0	1
8.0	Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.	1	1	0	1
10.0*	Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.	3	3	0	3
11.0	Students apply basic factoring techniques to second-and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect	1	1	0	1

California Content Standards		CDE CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
	squares of binomials.				
12.0*	Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.	1	1	0	1
14.0*	Students solve a quadratic equation by factoring or completing the square.	1	1	0	1
15.0	Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.	2	2	1	3
20.0*	Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.	1	1	0	1
22.0	Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points	0	0	1	1
23.0*	Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.	1	1	0	1
Geometry					
3.0*	Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.	1	0	0	0
4.0*	Students prove basic theorems involving congruence and similarity.	3	0	0	0
5.0	Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.	2	1	0	2
7.0*	Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.	2	2	0	2
8.0	Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figure.	1	1	4	5
9.0	Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.	1	1	0	1

California Content Standards		CDE CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
10.0	Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids	1	1	3	4
11.0	Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.	1	1	1	2
15.0	Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.	2	2	0	2
17.0	Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.	0	0	1	1
18.0*	Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.	2	0	0	0
19.0*	Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.	1	0	0	0
21.0*	Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.	2	2	0	2
22.0	Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.	0	0	1	1
Algebra II					
1.0*	Students solve equations and inequalities involving absolute value.	1	1	1	2
2.0*	Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.	3	3	1	4
3.0*	Students are adept at operations on polynomials, including long division.	1	1	0	1

California Content Standards		CDE CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
4.0*	Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.	1	1	0	1
6.0*	Students add, subtract, multiply, and divide complex numbers.	1	1	0	1
7.0*	Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.	2	2	0	2
8.0*	Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.	3	3	3	6
9.0*	Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x-b)^2 + c$.	0	0	1	1
10.0*	Students graph quadratic functions and determine the maxima, minima, and zeros of the function.	2	2	0	2
Standard Set 11.0* Students prove simple laws of logarithms.					
11.1*	Students understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.	1	0	0	0
12.0*	Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.	2	2	2	4
14.0	Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.	1	0	0	0
15.0*	Students determine whether a specific algebraic statement involving rational	1	0	0	0

California Content Standards		CDE CST Blueprint[a]	Reviewed CST Subset[b]	Reviewed Augment. Set[c]	Total Reviewed EAP[d]
	expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.				
17.0	Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.	0	0	1	1
18.0*	Students use fundamental counting principles to compute combinations and permutations.	1	1	0	1
19.0*	Students use combinations and permutations to compute probabilities.	1	0	0	0
22.0	Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.	1/2[e]	1	0	1
23.0*	Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.	1/2[e]	0	0	0
24.0	Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.	1	1	2	3
Probability and Statistics					
1.0	Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.	2	0	0	0
2.0	Students know the definition of *conditional probability* and use it to solve for probabilities in finite sample spaces.	2	0	0	0
7.0	Students compute the variance and the standard deviation of a distribution of data.	1	0	0	0

**Defined by the California Department of Education as a Key Standard. Key standards must make up a minimum of 70% of the CST.*

[a] *CSU EAP Test Committee is able to select from the full range of standards assessed on the CST*

[b] *Blueprint reflects items re-mapped during the analysis*

[c] *Achieve reviewers mapped the augmentation items to the California Content Standards. Some items address several standards.*

[d] *The final column includes the CST subset and the augmentation items.*

[e] *Fractional values indicate rotated standards (e.g., 1/2 = rotated every two years)*

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