

Quantitative Reasoning Courses

Quantitative Reasoning with Advanced Mathematical Topics and Transition to Quantitative Reasoning

A team based at Sacramento State University has developed two math courses designed for students who take a fourth-year quantitative reasoning course. Students choose these courses because they are interested in real-world application to mathematics. Both courses aim to offer students an option for math in the senior year that provides a strong foundation in quantitative reasoning.

Quantitative Reasoning with Advanced Mathematical Topics (QRAT) is designed for students who wish to challenge their math skills as they prepare to attend a UC or CSU but may not be ready for or interested in a pre-calculus/calculus pathway. The course meets the “C” (Mathematics) requirement for the University of California (UC) A–G admissions criteria. A passing grade in Integrated Math III or Algebra 2 is required to enroll in the course. QRAT builds on mathematical concepts learned in previous courses (including linear, quadratic, and exponential functions) to provide opportunities to develop a greater understanding of the underlying structures of mathematics and connections between mathematical topics.

Transition to Quantitative Reasoning (TQR) serves as an option for students who may not be interested in pursuing a STEM and/or math intensive major and may not typically take a math course in their senior year. The course fulfills the “G” elective of the UC A–G requirements. The units of study build upon previous math concepts including functions, inequalities, and exponents. The course is designed to encourage students to persevere through problem-solving and to develop quantitative reasoning skills needed in college and the workplace and may change their dispositions about mathematics.

Topics of Study in QRAT

- Developing Mathematical Practices
- Algebraic Foundations for Higher Mathematics
- Complex Numbers, Polar Coordinates and Vectors
- Polynomial and Rational Functions
- Introduction to Concepts of Calculus
- Inverse Functions
- Piecewise and Non-Invertible Functions
- Matrices and Linear Programming
- Economic Applications

Topics of Study in TQR

- Team Building and Problem-Solving
- Linear Functions
- Quadratic Functions
- Exponential Functions
- Logarithmic Functions
- Systems of Equations and Inequalities
- Absolute Value and Piecewise Functions
- Financial Mathematics

In the 2018–19 school year, either one or both of these courses were offered in 16 different school districts; by 2020–21, 20 different school districts were offering sections of one or both courses.

Who Enrolls in QRAT and TQR?

The two courses serve a student population that is generally reflective of the diversity of the cohort of 12th graders in the schools offering the courses:

QRAT



1,093

12th graders enrolled in QRAT in 2018–19
(13% of seniors in the schools offering the course)



36%

of enrollees identified as Latinx;

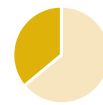
44%

of enrollees identified as white
(similar percentages of seniors in the cohort)



43%

of enrollees were designated as socioeconomically disadvantaged
(same percentage of seniors in the cohort)



36%

of 12th graders enrolled in QRAT *Met or Exceeded Standards* on math SBAC
(compared to 38% of seniors in the cohort)



81%

of 12th graders took Algebra 2/Integrated Math III the previous year

TQR



615

12th graders enrolled in TQR in 2018–19
(14% of seniors in the schools offering the course)



24%

of enrollees identified as Latinx;

48%

of enrollees identified as white
(compared to 25% and 42%, respectively, of seniors in the cohort)



37%

of enrollees were designated as socioeconomically disadvantaged
(compared to 42% of seniors in the cohort)



36%

of 12th graders enrolled in TQR *Met or Exceeded Standards* on math SBAC
(compared to 43% of seniors in the cohort)



72%

of 12th graders took Algebra 2/Integrated Math III the previous year

To better understand who the students are who enroll in these courses, it is helpful to understand the distribution of 12th graders across the different math courses. The figures that follow provide information on characteristics of twelfth grade math enrollment for all students in schools offering QRAT. Information on enrollment in various math courses is included for schools offering TQR, but additional descriptive data are not included in this brief as we focus on the “C”-approved QRAT course.

Table 1. 12th Grade Math Enrollment in Schools Offering QRAT

Math Course Category	Percent of 12th Graders Enrolled	
	2017–18	2018–19
Advanced Math Courses		
Calculus	14	14
AP Statistics	8	9
Statistics	4	4
Trigonometry, Precalculus & Other	13	13
QRAT	13	13
Algebra 2	13	15
Up to Algebra 2	10	11
No Math	31	27
Total 12th Graders	9,912	8,315

Note: Percentages do not add to 100% because some students enroll in more than one math course.

Table 2. 12th Grade Math Enrollment in Schools Offering TQR

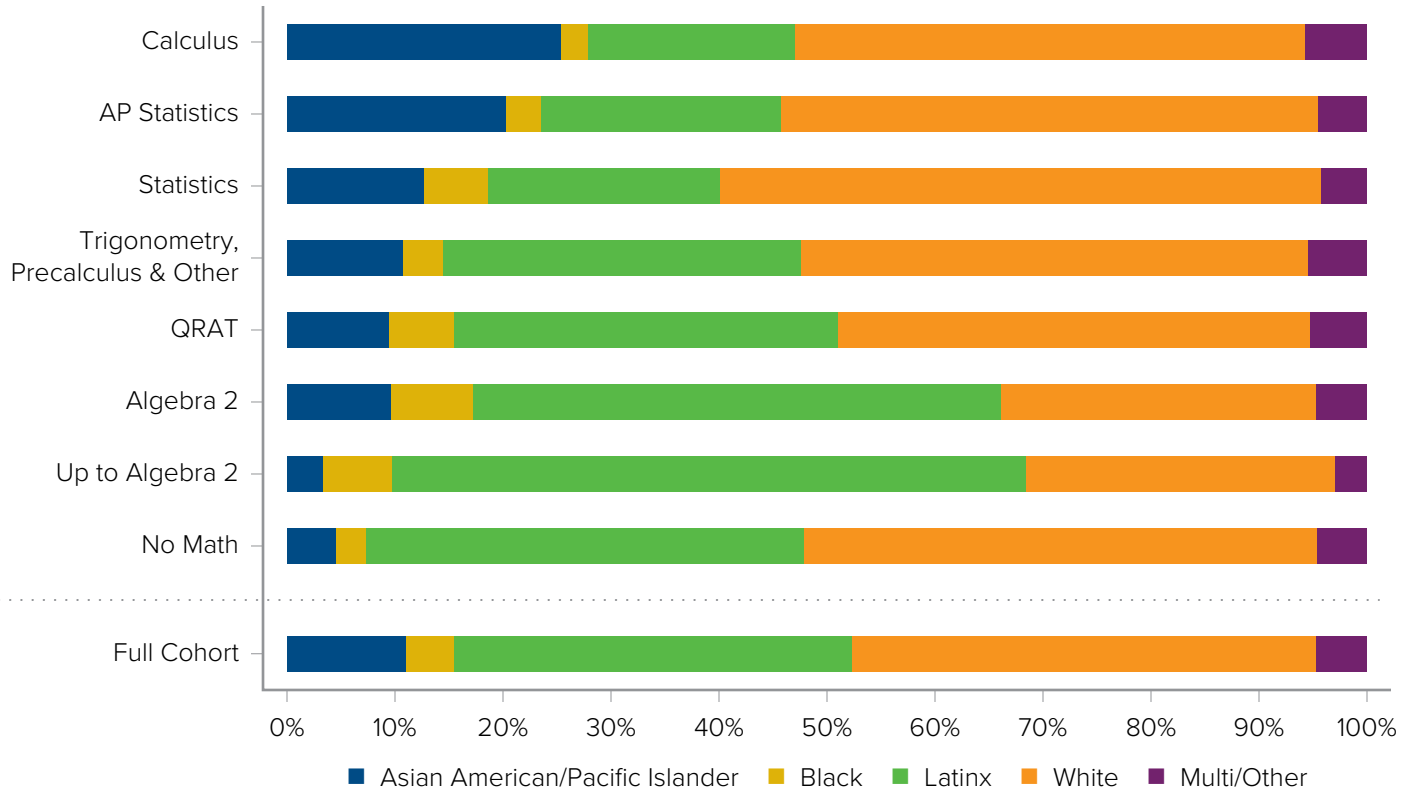
Math Course Category	Percent of 12th Graders Enrolled	
	2017–18	2018–19
Advanced Math Courses		
Calculus	8	14
AP Statistics	11	14
Statistics	0	0
Trigonometry, Precalculus & Other	14	25
TQR	14	14
Algebra 2	14	13
Up to Algebra 2	13	11
No Math	30	24
Total 12th Graders	2,776	4,488

Note: Percentages do not add to 100% because some students enroll in more than one math course.

DATA

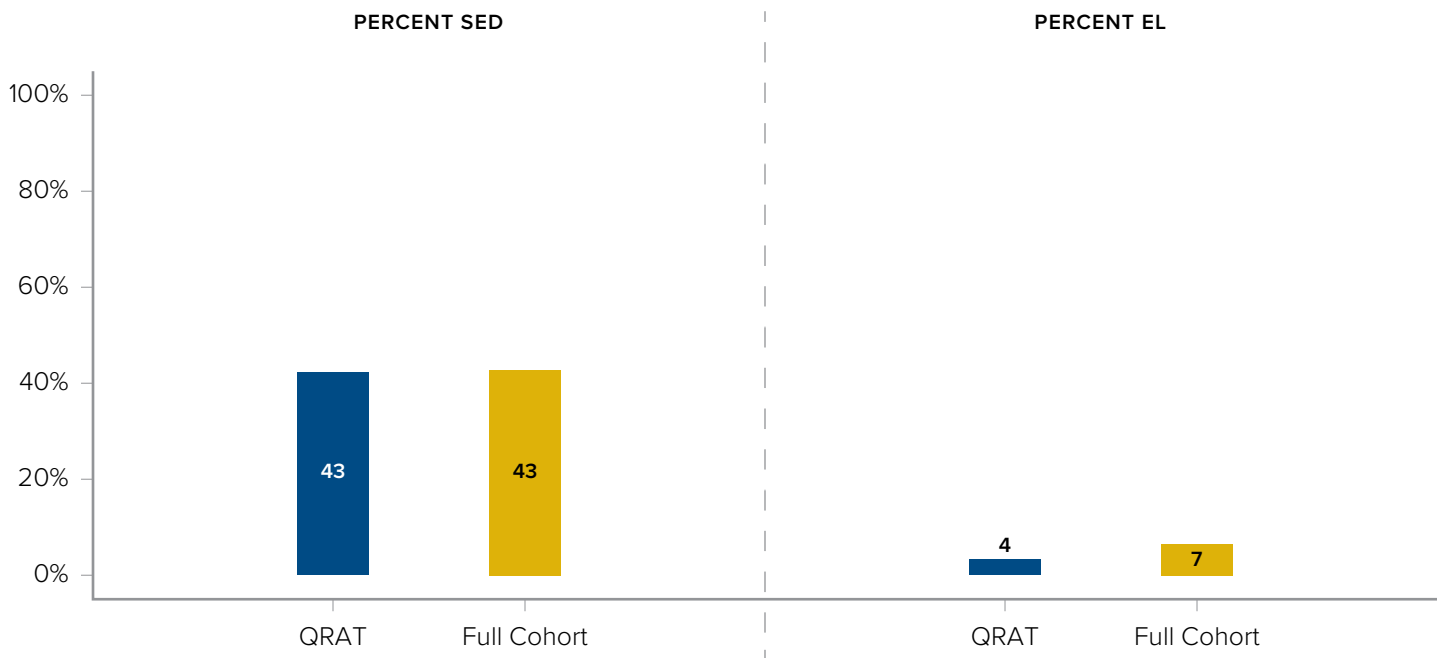
Statistics calculated from student-level and course-level data in the California Longitudinal Pupil Achievement Data System (CALPADS). Figures 1 through 3 include 12th graders in the 2018–19 academic year derived from participation in the 11th grade Smarter Balanced Assessments (SBAC) in 2017–18. Statistics may vary from other reports due to sample differences and data limitations.

Figure 1. Racial/Ethnic Composition of 12th Grade Math Courses in Schools Offering QRAT



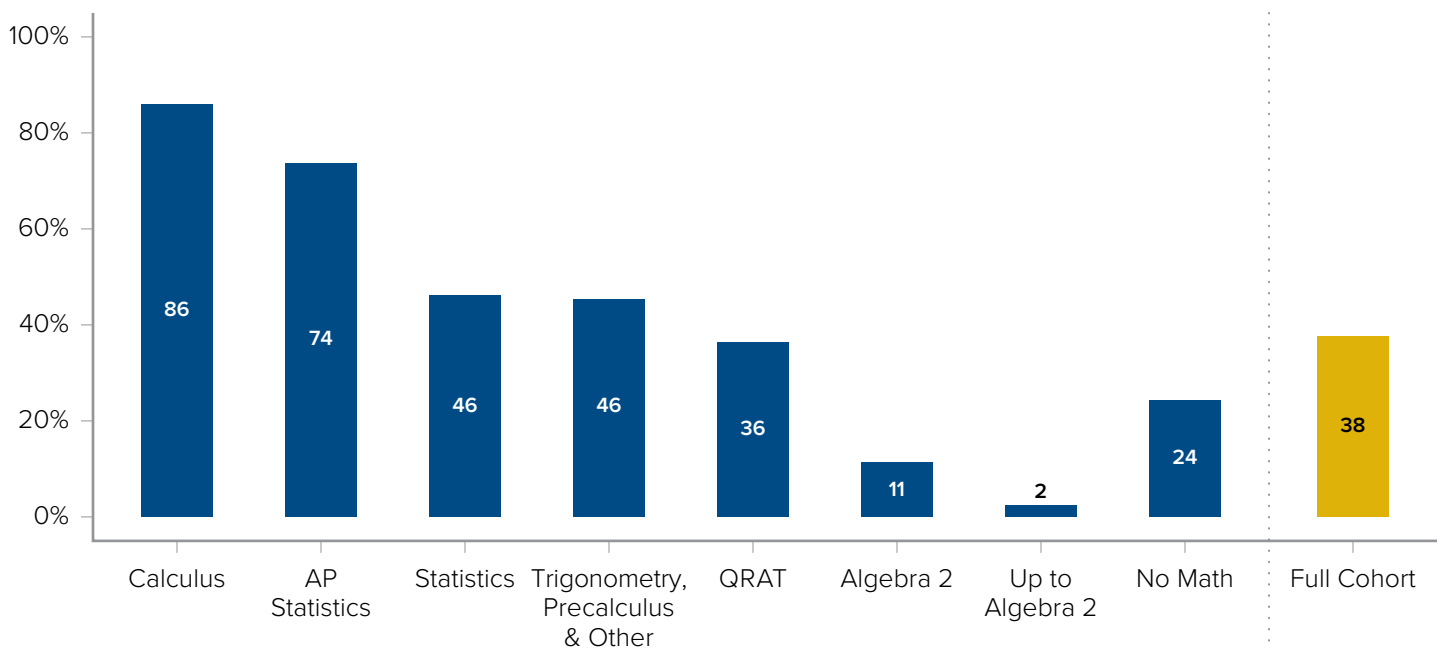
Note: Multi/Other includes students of Native American ancestry, students who identify with multiple racial/ethnic groups, and those for whom race/ethnicity information is missing.

Figure 2. Percent of SED and EL Students in 12th Grade QRAT Courses Compared to Full Cohort in Schools Offering QRAT



Notes: SED = Socioeconomically disadvantaged. EL = English Learner. SED status is defined by the California Department of Education as students who either qualify for the free or reduced-price school lunch program or do not have a parent who graduated from high school. For our analysis, we use the student-level SED and EL identifiers in the 11th grade SBAC data.

Figure 3. Percent of Students Meeting or Exceeding 11th Grade Standards on SBAC Math by 12th Grade Math Courses Enrollment in Schools Offering QRAT



Note: Students included in this figure achieved *Met Standard* (level 3) or *Exceeded Standard* (level 4) on the 11th grade math SBAC and were enrolled in high school courses the following year.

What Do Teachers Say about QRAT?

Teachers interviewed for this larger study were positive about the pedagogical approach used in these alternative math courses, noting that while it often takes a while for students to understand and appreciate the approach, they eventually become more comfortable and utilize skills that they have not previously been asked to use.

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**The pedagogy was very “guide on the side” and I really liked that.
My students did not. It took them a while to get used to it.
I kept answering their questions with a question and they hated it,
for lack of a better word, but they grew to understand why.**

(QRAT Teacher)

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Teachers cautioned, however, that just because the course can be enjoyable does not mean that it is not challenging. Due to the collaborative nature of the projects, the course requires attendance and participation much more so than many other courses:

“

**I would recommend [QRAT] for students that are serious about moving on to postsecondary education, just in terms of the work that needs to be put in.
It’s not a class where you can miss a week and take notes from a friend and kind of catch yourself up. You can’t go watch ten Khan Academy videos and be back up to par. This is a class where attendance is huge.
The ability to work in a collaborative environment is big.**

(QRAT Teacher)

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For more information on teachers’ experiences with QRAT, see our [full report on teacher perspectives](#).

What Is the Impact of QRAT for Students?

Analysis of QRAT and similar courses reveals a consistent positive story for students enrolled in advanced innovation math (AIM) curricula.

Leveraging high school course-taking data along with a rich set of student- and school-level characteristics linked to postsecondary enrollment data allowed for a causal analysis of the impact of enrollment in six AIM courses on key high school and postsecondary outcomes, including grades in 12th grade math courses, UC/CSU eligibility (completion of A–G course requirements), and college enrollment. Methods included advanced matching techniques to compare students enrolling in an AIM course to those who enroll in other (or no) math courses, controlling for a variety of factors that likely influence both math course placement and our outcomes of interest (e.g., prior student achievement in math as measured by test scores, previous math courses, demographic characteristics). Analyses for each AIM course were conducted respectively, and cannot be directly compared given unique course content, schooling environment, and the number of students and schools served. However, all courses demonstrate a positive impact for various postsecondary outcomes.

Results of these analyses indicate that AIM courses provide an additional math pathway that supports students' readiness for and, in some cases, increases in college enrollment. Specifically, AIM courses increase the likelihood that a student will meet the course requirements for UC/CSU eligibility by 3 to 10 percentage points. In some cases, AIM course enrollment improves grades in 12th grade math courses and increases the likelihood of college enrollment. Even when strong evidence of a positive impact on grades is absent, results are still promising. In fact, we may expect that rigorous curricula can engage and challenge students, which could result in a shift in grades, while increasing college preparation. Nevertheless, we are cautious about interpreting changes in grades given the many factors (i.e., teachers, peers, homework time, motivation) that influence grades and the observed effects of the AIM courses are small in magnitude. Moreover, there is limited potential for 12th grade math course performance to change college enrollment trajectories given fall due dates of four-year college applications.

Overall, early evidence of QRAT and similar courses is clearly promising: AIM courses contribute positively to student outcomes, offering students alternatives to traditional calculus pathways and increasing four-year college eligibility.

For more information on the development and implementation of QRAT and the other AIM courses, see our [full report](#).

AUTHORS AND ACKNOWLEDGEMENTS

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For more information on these courses or the statistics included in this report, contact Sherrie Reed at sreed@ucdavis.edu.