

# DEGREES OF FREEDOM: Diversifying Math Requirements for College Readiness and Graduation

(Report 1 of a  
3-part series)

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### EXECUTIVE SUMMARY

Since the mid-20th century, the standard U.S. high school and college math curriculum has been based on two years of algebra and a year of geometry, preparing students to take classes in pre-calculus followed by calculus. That pathway became solidified after the 1957 launch of the Soviet satellite Sputnik motivated reforms in U.S. science and engineering education to boost the nation's technological prowess. Students' math pursuits have been differentiated primarily by how far or how rapidly they proceed along a clearly defined trajectory that has changed little since then.

But evolutions in various disciplines and in learning sciences are calling into question the relevance and utility of this trajectory as a requirement for all students. The emerging movement is toward differentiated "math pathways" with distinct trajectories tied to students' goals. Alternatives emphasizing statistics, modeling, computer science, and quantitative reasoning that are cropping up in high schools and colleges are beginning to challenge the dominance of the familiar math sequence.

The drive toward acknowledging

the importance of multiple domains within math is prompted largely by two developments:

- **Technological tectonics.** The advent of new technologies is creating novel applications of math in various academic disciplines, elevating the importance of statistics, data analysis, modeling, and computer science in the undergraduate curriculum. It is also leading faculty members outside of math departments to pay more attention to their students' quantitative preparation.
- **Demand for deeper learning.** Learning scientists and math educators are emphasizing the importance of students' developing the capacity to use math skills and knowledge to solve problems in various contexts rather than simply learning isolated procedures and facts. American students' poor performance in traditional math sequences as well as the high proportion of college students taking remedial math have some reformers asking whether more applied courses would better lend themselves to the effective instruction needed to support college success.

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## EXECUTIVE SUMMARY (Cont.)

The dialogues converge in broader conversations about how colleges' expectations shape what students need to learn in K–12 schools—as well as in community colleges, which send hundreds of thousands of transfer students annually to four-year institutions. Decisions about math requirements and expectations will have a major impact on the academic opportunities of millions of students nationally.

### Technological Tectonics

Math and science associations are urging changes in undergraduate quantitative preparation to reflect changes already occurring in various disciplines. For example, at some research universities, biology professors are offering new math courses that emphasize statistics more than the traditional calculus-based course. Medical schools are revising entrance requirements to emphasize grounding in statistics. And social science departments such as sociology, psychology, and political science typically require statistics either for entering the major or completing a degree.

Students' college math enrollments are dictated by the demands of their intended major or by institution-wide graduation

requirements that became common over the last 50 years. As the requirements brought more students into math departments, remedial enrollments skyrocketed. Statistics enrollments also mushroomed, while enrollments in more standard courses, such as College Algebra, declined. Concern about math's relevance to students and responsiveness to other disciplines is sparking a number of reforms:

- promoting mathematicians' understanding of how math is used outside of math departments,
- redesigning College Algebra courses to emphasize modeling and other practical applications, and
- allowing alternative courses such as statistics and quantitative reasoning to fulfill quantitative graduation requirements.

### Demand for Deeper Learning

Discussions about improvements to pre-college math instruction are driven by alarm over weak achievement in math and the need to improve instruction in K–12 as well as college remedial courses. The vision of millions of college students spending time and money on high school material is unsettling to policymakers, parents, and students—even more so as research reveals that

remedial courses have no positive effects in terms of student success.

Nationally, about \$2 billion is spent on remedial math education. Among entering community college students, roughly 60 percent test into remedial math, the vast majority of whom never earn a college degree.

**Tension over Algebra 2.** Efforts to address this problem have diverged over the question of how much to emphasize Algebra 2, a challenging course that prepares students for Calculus. Algebra 2 became a fixture of high school curricula due to Ivy League entrance requirements, and a drive to offer it to all students grew from studies correlating the course with college success. While many agree about the value of Algebra 1 for a broad population of students, the necessity of Algebra 2 as a requirement for all, regardless of career plans or college major, has long been debated.

Concerns about college readiness have led some states to require Algebra 2 for high school graduation, while others are eliminating it to ensure that more students who are not pursuing science and technology-oriented majors can

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be considered college-ready (at least for community colleges).

The emerging trends in K–12 and community college math reform are moving in opposite directions in terms of addressing Algebra 2:

**Common Core implementation.** The Common Core math standards being implemented in California and most other states retain the traditional emphasis on two years of algebra and a year of geometry. Yet, they represent a fresh approach to math content within traditional topic lists because they emphasize “practice standards” that measure students’ mathematical maturity.

**Community college alternative remedial pathways.** The idea of alternative pathways is gaining particular traction as a strategy among leaders of efforts to improve remedial education and thereby increase graduation rates at community colleges. Fueled by foundation support and policymaker interest, experimentation is taking place in a number of states with alternatives that prepare students for statistics and quantitative reasoning. In 2014, the American Mathematics Associ-

ation of Two-Year Colleges (AM-ATYC) passed a resolution saying that intermediate algebra should not be a “universal prerequisite for all college-level mathematics courses.” The experiments have reported surprisingly strong results, tripling and quadrupling students’ passage of college-level math courses.

### Diversification and its Dilemmas

Given mathematics’ traditional prominence in the undergraduate curriculum, the implementation of alternatives faces a host of challenges:

- **Rigor.** Selective institutions that cannot admit all students often use rigorous courses as a screen to eliminate students. Community colleges and other broad-access public institutions, under pressure to increase graduation rates, are focused on rooting out unnecessary barriers and reject the idea of rigor for the sake of rigor. There is an inherent tension between the two perspectives.
- **Articulation.** Universities’ policies for admitting transfer students are also creating dilemmas for expanding alternative remedial pathways in some states, including California. Likewise, the success of the remedial courses, if it continues, could call into ques-

tion high schools’ continued reliance on more traditional math courses. But the idea of eliminating Algebra 2 gives many educators pause, since they fear this could prematurely close off opportunities for students.

- **Barriers.** While proponents of alternative pathways view algebra-intensive curricula as a potential barrier to students’ success, critics fear that an education without advanced algebra itself constitutes a barrier or dead end. One solution being explored is to make bridge courses available for students in alternative courses who discover an interest in STEM.

### Looking Ahead

California’s higher education institutions need to consider practical approaches that will best serve the students they have now, while simultaneously investing in and evaluating longer-term strategies to better prepare students for college-level courses.

The second and third reports in this series will include specific recommendations about policies and practices that do this. This report concludes with a set of principles

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that will underscore those specific recommendations:

- Keep the focus on maximizing student success statewide;
- At the college level, base remedial policies and other requirements on equipping students for subsequent coursework, career, and citizenship (rather than abstract notions about desired knowledge);
- Increase communication so segments can better understand the implications of their policies on students in other segments and minimize misalignment;
- Apply greater scrutiny to academic requirements noted for high failure rates; and
- Make room for experimentation and innovation while gathering evidence.

# DEGREES OF FREEDOM: Diversifying Math Requirements for College Readiness and Graduation

## Foreword

Since the mid-20th century, the standard U.S. high school and college math curriculum has been based on two years of algebra and a year of geometry, preparing students to take classes in pre-calculus and calculus. That pathway originated with admissions requirements at Harvard and other elite universities. It became solidified after the launch of the Soviet satellite Sputnik motivated reforms in U.S. science and engineering education to boost the nation's technological prowess. Since then, it has effectively been a matter of faith that the calculus-directed sequence of courses is essential content for college success.

Students' math pursuits, then, have been differentiated primarily by how far or how rapidly they proceed along a clearly defined trajectory that has changed little since the 1950s. While the traditional course pattern remains the norm, evolutions in various disciplines and in learning sciences are calling into question its relevance and utility as a requirement for all students. Alternatives emphasizing statistics, modeling, computer science, and quantitative reasoning are cropping up in

high schools and colleges and beginning to challenge the dominance of the familiar sequence. The emerging movement is toward differentiated math pathways with distinct trajectories tied to students' goals. Consider these examples:

- In Texas, instead of an Algebra 2 graduation requirement, high school students can now opt to take courses in statistics or algebraic reasoning.
- In Los Angeles, a \$12.5 million National Science Foundation grant is allowing some high schools to pilot a data science class intended to replace Algebra 2 for some students.
- At about 50 institutions (mostly community colleges) in 14 states, the Carnegie Foundation for the Advancement of Teaching is testing new approaches to remedial math that emphasize statistics and quantitative reasoning rather than the traditional algebra-intensive curriculum.
- In Georgia, a state-level task force concluded that college students who aren't pursuing majors in science and technology may ful-

fill their math requirements with courses called Quantitative Skills or Introduction to Mathematical Modeling instead of College Algebra.

- And at the University of California’s Los Angeles and Berkeley campuses, life sciences professors are offering new math courses that emphasize statistics more than the calculus-based class traditionally required for biology majors.

Proponents of such diversity say it better reflects the evolving role of math outside of math departments. “Math has grown incredibly in the last 25 years,” notes Uri Treisman, a mathematician at the University of Texas at Austin and leader of several math reform initiatives. “It’s not just areas of engineering, although they remain extremely important. It’s Pixar, Netflix, Google, Facebook, tomography, epidemiology, mathematical pharmacology, machine learning. If you look at the mathematics we have in the curriculum now, it represents one powerful strand among a forest of powerful strands of mathematics.”

The drive toward acknowledging the importance of multiple domains within math is being played out in schools and colleges in California and nationally, prompted by two developments:

- **Technological tectonics.** The advent of new technologies is creating novel applications of math in various academic disciplines, elevating the importance of statistics, data analysis, modeling, and computer science in the undergraduate curriculum. It is also leading faculty members outside of math departments to pay more attention to their students’ quantitative preparation. Concerned about ensuring the relevance of their field, mathematics societies are encouraging greater dialogue between math departments and other disciplines that use math.
- **Demand for deeper learning.** Learning scientists and math educators are increasingly emphasizing the importance of students’ understanding mathematical concepts and developing the capacity to use math skills to solve problems in various contexts rather than simply learning isolated procedures and facts. Research suggests that the de-contextualized instruction common in American math classrooms can actually impede learning (Stigler & Hiebert, 1999; Grubb, 2013). Some suspect it can even deter students from pursuing scientific fields (President’s Council, 2012). Some educators, researchers, and policymakers are asking whether more applied courses—such as those emphasizing statistics and

quantitative reasoning—would better lend themselves to the effective instruction needed to support college success.

“When today’s parents were going through the schools, the main focus in mathematics was on mastery of a collection of standard procedures for solving well-defined problems that have unique right answers. Learning mathematics had been that way for several thousand years. Math textbooks were essentially recipe books,” Stanford mathematics professor Keith Devlin told *Forbes Magazine*.

“Now all those math recipes have been coded into devices, some of which we carry around in our pockets. Suddenly, in a single generation, mastery of the procedural math skills that had ruled supreme for 3,000 years has become largely irrelevant. Students don’t need to train themselves to do long computations, as was necessary when I was a child. No one calculates that way any more! What they (we) need in today’s world is a deeper understanding of how and why Hindu-Arabic arithmetic works,” he said.

The two dialogues have arisen in different realms: The technological shifts pertain to undergraduate education, while the learning demands relate more directly to how students are prepared for college, career, and

general citizenship. They converge in broader conversations about how colleges' expectations shape what students need to learn in K–12 schools—as well as in community colleges, which send hundreds of thousands of transfer students annually to four-year institutions.

Will universities, for example, admit high school students who took a data science class in lieu of Algebra 2? Will community college students taking the Carnegie Foundation's Statway sequence instead of a traditional math sequence be able to transfer to the university of their choice? If so, what majors will be open to them? Will the biology faculty interest in statistics training ultimately influence quantitative expectations for incoming freshmen? And how will the new Common Core State Standards being implemented in most states affect those choices?

These decisions and the interactions among them will have a major impact on the academic opportunities of millions of students nationally. And yet, discussions about them have occurred largely in isolation, with each segment of the education system bringing different priorities to bear. Interestingly, the increasing engagement of researchers and policymakers in improving student

outcomes in community colleges is helping to make the connections more explicit. That's because community colleges sit at the nexus of the two conversations, offering remedial courses that review K–12 material as well as lower-division courses that transfer to four-year institutions.

LearningWorks' 2013 report, *Changing Equations: How Community Colleges Are Re-Thinking College Readiness in Math*, focused on experiments with alternative curricula at two-year colleges. That report noted that those efforts were not in sync with math requirements in high schools and four-year universities. Given that these, too, are in flux, LearningWorks commissioned an additional series of reports to analyze the changing status of math requirements throughout the education system, its implications for students' progress from high school through college, and the role of schools, community colleges, and universities in supporting such progress.

The series, *Degrees of Freedom*, is based on dozens of interviews and extensive reviews of literature and documents related to U.S. high school and college math curricula since the mid-1900s. This report will examine the move toward differentiated

math pathways linked to students' academic majors, highlighting some obstacles to implementing them and some principles for addressing those obstacles.

The rest of the series will focus more on math-related transitions in California's education system and surface more specific policy recommendations for improving them. The second report will analyze how the distinct missions of high schools, community colleges, and four-year universities create tensions for articulating math curricula across segments, and the resulting challenges for alternative math pathways as a route to college readiness. The third report will analyze more specifically how college and university policies for placing students in remedial math courses create additional barriers for students in completing a degree or transferring to a four-year university.

*Degrees of Freedom* is being disseminated by LearningWorks and PACE (Policy Analysis for California Education) with a goal of fostering needed conversations spanning K–16 education to consider strategies to enhance student success in math and improve curricular coherence for students as they progress through the educational system.

## TECHNOLOGICAL TECTONICS AND MOMENTUM FOR CHANGE

Undergraduate math curricula are undergoing change nationally. College math faculties are facing pressure to transform their course offerings and make them responsive to the needs of other disciplines. In 2014, the Joint Policy Board for Mathematics (an umbrella organization of several math and statistics societies) issued a statement “calling upon the entire mathematical sciences community to achieve much-needed change in undergraduate education.” And the National Research Council’s 2013 report, *Mathematical Sciences in 2025*, noted, “The need to create a truly compelling menu of creatively taught lower-division courses in the mathematical sciences tailored to the needs of twenty-first century students is pressing.” (See Box, *National Associations Urge Change*, p. 8.)

Many of the changes remain experimental and peripheral, if not controversial. While there is broad agreement that new ways of teaching math are needed, some experts, including some mathematicians, are cautious about deviating from conventional math content. Leaders in higher education, wary of creating pathways that could be considered

second-tier, tend to defer to math departments when it comes to setting standards. More fundamental doubts exist about how much weight fields such as statistics and computer science deserve. “A lot of mathematicians don’t consider statistics a reasonable part of mathematics,” notes Jeremy Kilpatrick, a math education expert at the University of Georgia. “For them, it’s not really mathematics. It’s an application of mathematics, but not mathematics itself.”

Controversy aside, there is clear momentum for diversifying students’ quantitative preparation. If it continues, the revised courses could grow to constitute a dominant proportion of pre-collegiate and college-level math courses in years to come. They are already gaining a foothold in some areas of the undergraduate curriculum. Since math requirements are typically intended to prepare students for later coursework, this examination of the shifts begins by looking at the evolution of math requirements for undergraduate majors.

### Life Sciences and Math Requirements

Take the re-design of math requirements in some biology departments. At both UC Berkeley and UCLA, the traditional one-year calculus course may eventually be replaced by a course combining calculus and

statistics as the required math course for biology majors. At Berkeley, the new class—entitled *Methods of Mathematics: Calculus, Statistics and Combinatorics*—was created by a committee of mathematicians and biologists. While some mathematicians were initially skeptical, it helped that the biology professor behind the class had a joint appointment in mathematics. He ensured his colleagues that students would still be studying advanced math.

“Molecular biology in particular has become a mathematical science,” said the professor, Lior Pachter. “The real radical change is happening with DNA sequencing. There’s a lot of data collection and statistics.” His course now enrolls about 300 students, primarily biology majors.

At UCLA, the process has been more controversial. About four years ago, Life Sciences faculty dissatisfied with their students’ math preparation created the specifications for a new course. While de-emphasizing some typical introductory calculus topics, *Mathematics for Life Scientists* stresses topics such as statistics, dynamic modeling, and differential equations. It also includes computational labs and increased teaching assistant involvement. Unlike the traditional sequence of three trimesters of Calculus plus a fourth of Statistics, the new sequence consists of three trimesters total. The math-

## NATIONAL ASSOCIATIONS URGE CHANGE

In the last five years, a host of math and science associations have urged changes in undergraduate quantitative preparation:

### **2014—Joint Policy Board for Mathematics, *Meeting the Challenges of Improved Post-Secondary Education in the Mathematical Sciences***

“We call upon all mathematical scientists in academia to renew their focus on post-secondary mathematics education. We challenge department chairs to incentivize innovation for the sake of their students and the health of our discipline. We encourage mathematics faculty to reach out to colleagues in mathematics-intensive disciplines in order to heighten the relevance of their courses to the careers of their students. And we urge departments as a whole to investigate with an open mind new teaching methodologies and technologies, keeping in mind the need to retain and motivate students.”

### **2013—National Research Council, *Mathematical Sciences in 2025***

“The educational offerings of typical departments in the mathematical sciences have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large. This diversification entails a need for new courses, new majors, new programs, and new educational ... partnerships ... Different pathways are needed for students who may go on to work in bioinformatics, ecology, medicine, computing, and so on. It is not enough to rearrange existing courses to create alternative curricula.”

### **2012—President’s Council of Advisors on Science and Technology, *Engage to Excel***

“Employers in the private sector, government, and military frequently cite that they cannot find enough employees with needed levels of mathematics skills. This lack of preparation imposes a large burden on higher education and employers.

Higher education alone spends at least \$2 billion per year on developmental education to compensate for deficiencies. Also, introductory mathematics courses often leave students with the impression that all STEM fields are dull and unimaginative, which has particularly harmful effects for students who later become K–12 teachers. Reducing or eliminating the mathematics-preparation gap is one of the most urgent challenges—and promising opportunities—in preparing the workforce of the 21st century.”

ematics department, however, did not find the math content acceptable, so the College of Life Sciences began its own pilot in 2013, supported by a \$2.3 million National

Science Foundation grant to develop and evaluate the course.

Changes in these and other biology departments correspond with a shift starting to happen at medi-

cal schools. For decades, Harvard Medical School, for example, asked incoming students to have at least a year of calculus under their belts. But 2015 will be the final year that a stan-

dard Calculus course will be accepted. The school's revised requirement applies greater emphasis on statistics than on calculus, which it says often focuses on the "derivation of biologically low-relevance theorems." Besides a "familiarity with calculus," it encourages a "broader and more flexible range of requirements," adding that, "given the importance of statistics for understanding the literature of science and medicine, adequate grounding in statistics is required."

Other leading medical schools such as Johns Hopkins and Stanford are moving in a similar direction, stressing grounding in statistics, epidemiology, and the capacity to evaluate scientific research reports. Such changes echo recommendations the American Association of Medical Colleges made in 2009.

Since some critiques of the traditional math curriculum have come from individuals in the humanities and social sciences, their proposals are sometimes eyed suspiciously as sacrificing quantitative rigor. The involvement of prestigious biology departments and medical schools in re-thinking math requirements could lend legitimacy to alternative approaches and give rise to a new definition of mathematical rigor. Its goal would be to ensure that U.S. institutions foster technological prowess and attract talented students to technical fields while simultaneously

## LEADING MEDICAL ORGANIZATIONS REVISE THEIR MATH PREREQUISITES FOR PHYSICIANS

### 2010—Harvard Medical School:

*A full year of calculus focusing on the derivation of biologically low-relevance theorems is less important than mastery of more relevant algebraic and trigonometric quantitative skills. To prepare adequately for the quantitative reasoning demands of the contemporary medical curriculum and certain medical specialties, to provide analytic perspective and to appreciate the uncertainties in evaluation of biological systems, students are required to have familiarity with calculus. A broader and more flexible range of requirements is encouraged, however, and given the importance of statistics for understanding the literature of science and medicine, adequate grounding in statistics is required.*

### 2009—Scientific Foundations for Future Physicians—Entering Medical Student Expectations

*Learning Objectives:*

1. *Demonstrate quantitative numeracy and facility with the language of mathematics.*
2. *Interpret data sets and communicate those interpretations using visual and other appropriate tools.*
3. *Make statistical inferences from data sets.*
4. *Extract relevant information from large data sets.*
5. *Make inferences about natural phenomena using mathematical models.*
6. *Apply algorithmic approaches and principles of logic (including the distinction between cause/effect and association) to problem solving.*
7. *Quantify and interpret changes in dynamical systems.*

developing the quantitative abilities of students pursuing non-technical fields. A tall order, to be sure.

### Quantitative Graduation Requirements and the Growth of Statistics

Elsewhere in the undergraduate curriculum, students' math enroll-

ments are generally dictated either by the demands of their intended major or, for those whose majors don't specify math expectations, by university-wide general education requirements. University-wide math expectations weren't always a fixture of graduation requirements. In the 1960s, according to a Mathemati-

cal Association of America (MAA) report, the organization approached the idea of quantitative literacy for all students in a “gingerly manner” (MAA, 1994). At the time, a low point for general education, a small minority of institutions required math for all students. In the early 1970s, the vast majority of math department chairs surveyed by the Conference Board of the Mathematical Sciences (CBMS) did not think institution-wide requirements were even necessary.

The requirements began gaining prevalence in the 1970s, along with the expansion of general education, leading to math’s current “gatekeeper” status within the general education curriculum. This growth coincided with enrollment expansion and demographic change. But there was a tension between the mathematicians’ view of their discipline and the idea of general education.

“When it came to having general education requirements, history and psychology departments developed service courses for non-history and non-psychology majors, for example. Mathematics didn’t do that. They said, ‘We’ll teach them the same thing we’re teaching now,’” noted Bernard Madison, a University of Arkansas mathematician and math educator. “Mathematicians never really came to grips with using mathematics in service of general education.”

Meanwhile, a parallel shift was occurring: As overall enrollment in mathematical sciences was increasing significantly, much of the growth was in computer science and statistics courses, not mathematics itself. Upper division math enrollments were actually dropping—falling by 32 percent from 1970 to 1975, for example. “The clear overall impression from course enrollment data is a shift toward mathematical science courses that are applicable as preparation for specific post-college careers,” noted the CBMS in its 1980 survey.

At the same time, remedial enrollments were jumping dramatically, more than doubling in a decade at both two-year and four-year colleges. While noting that the increase matched professors’ impressions of declining preparation levels among students, the CBMS reports do not discuss the likelihood that quantitative graduation requirements (paired with overall enrollment growth in higher education) were responsible for bringing more under-prepared students into math departments. Math requirements, then, became a *de facto* filter that determined students’ readiness for college.

In more recent years, discussions have centered not on whether to have quantitative graduation requirements—most higher education institutions do—but on how to

make them relevant and engaging for students. Today the majority of students appear to meet them with a course situated in between Algebra 2 and Calculus—most often College Algebra or Pre-Calculus. (While the definitions of these courses differ among institutions, and there is significant overlap between them, typically Pre-Calculus includes trigonometry.)

But other mathematical sciences have seen dramatic increases in enrollment—particularly statistics. In 1960, the first year for which data is available, statistics represented about three percent of all mathematical science enrollments at four-year institutions. By 2010, 19 percent of introductory math enrollments at four-year schools were in statistics courses, according to the CBMS survey. College Algebra and similar courses, while still dominant, constituted a shrinking proportion of introductory enrollments from 1995 to 2000, even as enrollment in other introductory courses (including Liberal Arts Math, Finite Math, Business Math, and Mathematics for Teachers) grew by 37 percent.

A different 2010 survey of math departments found that about three quarters of colleges allowed students to meet quantitative graduation requirements with Statistics, and half allowed courses outside of math departments. Seventeen percent al-

lowed a course focused specifically on quantitative reasoning or quantitative literacy (Schild, 2010).

Ironically, then, the overall expansion of math enrollments coincided with the adoption of quantitative graduation requirements, but alternatives such as Statistics have grown disproportionately, as institutions looked for courses that were palatable to students outside of math-intensive majors. Statistics is said to appeal to students with strong conceptual capacity whether or not they excel at algebraic manipulation. The increasing demand for “data crunchers” in fields ranging from biomedical research to consumer marketing to policy analysis led Google’s chief economist to call data crunching a “sexy job.” And AP Statistics has grown from 7,000 exams in 1997 (compared with 110,000 in Calculus) to 180,000 exams in 2014 (compared with 280,000 in Calculus).

Statistics also plays a key role in many social sciences. Researchers in psychology, sociology, and political science increasingly employ data collected by hospitals, schools, social service agencies, and other public entities. Many social science departments require statistics courses. Some offer one for their majors, presenting statistical techniques within the context of the discipline in order to familiarize students with

data sources and research methodologies common in their fields. Such courses fulfill general education requirements at some institutions, but others require College Algebra as a general education course and Statistics as an additional, upper-division course.

### **Interdisciplinary Discussions and Math Requirements**

Aware of math’s gatekeeper role, the MAA has actively cultivated the involvement of faculty from the social sciences and other disciplines in conversations about students’ quantitative preparation. This, in turn, has led to shifts in priorities. In 2007, when the organization brought social scientists and mathematicians together to develop recommendations, the participants felt that college students ideally should study both advanced algebra and statistics. But noting the impracticality of a two-course sequence, they concluded that it is “imperative that students come away with an appreciation for statistics, and that this preparation takes priority over more typical introductory mathematics courses” (Ganter & Haver, 2011).

With the decline in math majors, and an increasing number of quantitative requirements being fulfilled in other departments, some math leaders are concerned about staying relevant. “I think that it needs

to be realized that mathematics, as taught by mathematicians, is seen as a separate and arcane discipline by a large fraction of the academic community and that this is a problem for our profession,” wrote one math chair in responding to a 2010 survey about math requirements (Schild, 2010).

In fact, cultivating mathematicians’ understanding of how math is used outside of math departments has been seen as vital to the future of mathematics. “It’s a real problem, because many mathematicians and teachers of mathematics are not aware of all the new and different ways in which math is now being used,” noted Mark Green, a retired UCLA math professor and lead author of the *Mathematical Sciences in 2025* report.

Devlin recalls coming to Stanford in 1987, expecting to benefit from the institution’s prowess in technology research and discovering something broader. “What changed me far more was coming up against the world-class expertise in the human sciences that Stanford also has,” he told Forbes. “It was after working with (actually, mostly listening to) leading human scientists and philosophers who consider those issues that I was forced to reassess the role that mathematics can play in the human sciences.”

## QUANTITATIVE COURSE SELECTION: COURSES FULFILLING QUANTITATIVE REASONING REQUIREMENTS AT SIX UNIVERSITIES

### Stanford

- Cancer Epidemiology
- Riding the Data Wave
- Electric Automobiles and Aircraft
- Experimental Economics
- Remote Sensing of the Oceans
- Feeding Nine Billion

### Harvard

- Making Sense: Language, Logic, and Communication
- Analyzing Politics
- Deductive Logic
- Nutrition and Health
- Myths, Paradigms, and Science
- Great Ideas in Computer Science

### UCLA

- Biostatistics
- Computing
- Inductive Logic (Philosophy)
- Introduction to Data Analysis (Political Science)
- Statistics

### San Francisco State University

- Calculus with Business Applications
- Data Analysis in Education
- Calculus or Business Calculus
- Elementary Statistics
- Pre-Calculus
- Quantitative Reasoning in Psychology

### Cal State University- Northridge

- College Algebra
- Mathematical Methods for Business
- Pre-Calculus
- Mathematical Ideas
- Introductory Statistics
- Calculus for the Life Sciences

### CUNY- College of Staten Island

- Mathematics for Liberal Arts
- Finite Mathematics
- Probability and Statistics
- College Algebra and Trigonometry
- Pre-Calculus

Source: Institutions' websites. With the exception of San Francisco State University, the courses listed are a subset of those accepted to meet quantitative reasoning requirements.

This increasing involvement of other disciplines in discussions about math may be eroding mathematicians' traditional veto power over quantitative requirements. "People in other departments are really questioning the learning goals, and saying, 'Those shouldn't be our learning goals.' That suddenly is taking the power away from the math departments," notes James Stigler,

a UCLA psychologist who studies math learning. "The questioning of the idea that the math department knows what math all students need to know is new."

### College Algebra and Curricular Change

Despite gains by Statistics and other alternatives, the traditional courses

still represent the majority of introductory math enrollments, largely due to general education requirements. College Algebra courses enroll close to a million students per year in two-year and four-year colleges combined—with two-year enrollment having quadrupled over the last 40 years. Such large enrollments have historically been a source of math departments' influence.

College Algebra's role as a staple of the undergraduate curriculum is sometimes compared to Latin, which fell out of favor in the last century. "Many would skip College Algebra if they did not have to pass it to get the degree they need to enter their chosen career," according to a 2002 address by Arnold Packer, a former Department of Labor official who directed a federal commission on 21st century skills in the workforce, "For too many students it looks like—and is—a painful experience that they would prefer to skip" (Packer, 2002). Packer was speaking at an MAA conference about College Algebra. Conference participants cited failure rates above 40 percent as evidence that the course needed reform, particularly since the educational purpose of the large enrollment is unclear: Though College Algebra is intended to be a prerequisite for Calculus, fewer than 10 percent of College Algebra students ultimately take Calculus (Small, 2002).

Some of the interdisciplinary discussions fostered by the MAA have focused on ways of modernizing the course. A key reform is an emphasis on modeling, which an MAA report says students find more relevant and engaging than the standard version. "Generally, attendance and the classroom environment improve dramatically. Most importantly, in our view, students develop habits of

mind necessary for life-long learning while learning the mathematics that will be most important to them in future endeavors," the report said (Ganter & Haver, 2011).

As reform efforts accelerate, it's become clear that a reason College Algebra enrollments remained high in some places has been the absence of other options for meeting quantitative requirements. "It's about traditional views of rigor and not so much about relevance," notes Bruce Vandal, vice president of Complete College America, an advocacy group that promotes alternative math pathways as a means of increasing college completion rates.

That is beginning to change, however. For example, by 2010, Arizona State University had removed College Algebra from the list of courses that fulfill its quantitative graduation requirement. "The department has taken this action as it believes students requiring only one mathematics course in their college experience should be introduced to mathematics that is more applied in nature," according to a survey response. "We further believe any student taking College Algebra should have every intention of taking another mathematics course" (Schield, 2010).

In Georgia, a task force including respected mathematicians reached a similar conclusion in 2013: "The

practice of using College Algebra as a proxy for general quantitative ability or to ensure that students can later transfer to a STEM major must end," it wrote. "System data suggest that placement in College Algebra for the above reasons is a major contributor to student failure. Furthermore, the broad audience in College Algebra makes it challenging to organize this important course as a true stepping stone to Calculus" (University System of Georgia, 2013). The task force recommended that students who aren't majoring in STEM fields should be allowed to complete their math requirements with courses in quantitative skills or mathematical modeling.

Interestingly, diversity in math requirements appears to be more common at selective institutions, with broader access universities and colleges tending to have narrower offerings. Examples of acceptable courses at Stanford University include: Feeding Nine Billion, Remote Sensing of the Oceans, and Cancer Epidemiology. Harvard accepts courses including Making Sense: Language, Logic, and Communication; Deductive Logic; Nutrition and Health; and Myths, Paradigms, and Science (see Box, *Quantitative Course Selection*).

As with the innovations occurring in biology departments and elsewhere in universities, however, there

are few avenues for sharing and disseminating information about these courses. An unpublished survey in 2010 appears to be the best overview available of how institutions treat their quantitative requirements. Therefore, it is unknown how many institutions offer such a broad range of options, and for those that do, to what extent students transferring from two-year colleges can use them to meet math requirements.

“Almost everywhere that people have capacity and are actively engaged in mathematical research and have people who care about undergraduate education, there’s experimentation going on,” notes Treisman. “The problem isn’t a lack of experimentation. It’s that the experimentation takes place in a fog of collective amnesia.” Treisman, Green, and other leading mathematicians have initiated a project called TPSE Math (Transforming Postsecondary Education in Math) focused on the modernization of the undergraduate math curriculum. Undoubtedly, one of their tasks will be to examine the current state of experimentation and innovation in the general education curriculum.

### **LOW MATH ACHIEVEMENT AND THE DEMAND FOR DEEPER LEARNING**

The momentum for diversifying postsecondary math curricula stems

from the quantitative and statistical demands of various academic disciplines and industries and the ineffectiveness of courses such as College Algebra in preparing students for these demands. Discussions about improvements to pre-college math instruction are commonly driven by alarm about U.S. students’ weak achievement in math. As such, they are often disconnected from discussions about the undergraduate curriculum for which they are intended to prepare students. Yet these discussions, too, have begun to focus on the possibility that alternative math content could actually provide a better foundation for some students.

The reasons for poor math achievement have been explored in debates that unfortunately are best known for their lack of consensus. The evidence—test scores—has been less controversial. On the most recent PISA (Programme for International Student Assessment) round, for example, U.S. students ranked 26th out of 34 countries in math. Even against domestic standards, U.S. students don’t look much better, with only seven percent of 17-year-olds reaching the highest level of proficiency on standardized tests.

In recent years, though, the most glaring evidence of students’ math deficiencies has been the large proportion who are sent to remedial math courses upon taking college place-

ment exams. The vision of millions of college students spending time and money on high school material is an unsettling one to policymakers, parents, and students alike—even more so as research has revealed that these courses have no positive effects in terms of student success.

Nationally, about \$2 billion is spent on remedial math education (President’s Council, 2012). About 35 percent of students attending four-year universities require a remedial course in English or math, and among entering community college students, around 59 percent test into remedial math. The vast majority of those students never earn a college degree (Hodara, 2013).

“The developmental math problem is an indictment of our whole education system,” notes Stigler of UCLA. There is general agreement that vast improvement in instruction is needed in both K12 and remedial math classes. In the math education community, there is an emerging consensus that students lack conceptual understanding, which is directly attributable to the way mathematics is taught.

“The major problem is that the way math has been cut up, chopped up, and delivered makes it hard to get sense-making,” notes Alan Schoenfeld, a professor of education at UC Berkeley. “What kids get, by and

large, is a diet of facts and formulas, which they can remember with cramming for the final exam, and by the time September rolls around, they've lost half of it."

At one Southern California community college, Stigler and colleagues found that only 22 percent of students who took the math placement exam could successfully order four numbers consisting of two simple fractions and two decimals. After converting the fractions to decimals, sixty percent of students ordered them by the number of digits.

"These errors reveal that rather than using number sense, students rely on a memorized procedure, only to carry out the procedure incorrectly or inappropriately," they said (Stigler et al., 2010). The findings echoed Stigler's earlier research showing that, compared to students in higher performing nations, U.S. students tended to learn math as a set of formulas without understanding how to think mathematically, and that this won't change without different teaching methods (Stigler & Hiebert, 1999).

### Emerging Strategies and Tension Over Algebra 2

The conclusion of this body of research is that students must move beyond discrete skills and formulas used to answer test questions toward a conceptual understanding in which they are able to reason and solve

problems and nimbly apply those skills in other contexts. From a content perspective, efforts to address this challenge lie on a continuum whose extremes are:

- **New approaches to existing content:** These entail finding ways to teach the traditional content more effectively so that more students succeed. Many efforts have adopted this approach, particularly because of the imperative of preparing a subset of students to succeed in STEM fields. To date, there has been limited success, however. High failure rates in high school as well as college remedial math classes persist, impeding some students from earning college degrees. These approaches are most common in high school, where the philosophy has been to keep students' options open.
- **New math content:** Another approach is to develop math content that may be more relevant for students, particularly those who are not pursuing STEM fields, and that better supports their acquisition of quantitative reasoning skills. Diversification in the college math courses taken by non-STEM majors lends more credibility to this approach. This work is being pursued mainly in community colleges.

Few efforts lie at one end or the other. Indeed, there is a strong argument that both improved instructional approaches as well as updated curricula are needed. However, the dominant K–12 and community college math reforms are moving in opposite directions in their treatment of Algebra 2. While K–12 schools implement Common Core-aligned curricula that retain an algebra emphasis, alternative remedial pathways that downplay second-year algebra are gaining steam in community colleges.

Many agree about the value of Algebra 1 for a broad population of students. But whether Algebra 2 should be required for all students, regardless of educational or career aspirations, has long been debated. The course became a fixture of high school curricula due to Ivy League entrance requirements in the 1950s and a drive to offer it to all students grew from studies correlating the course with college success (Achieve Inc., 2004).

Algebra 1 (often called Elementary Algebra at the remedial level) introduces the basic concept of using variables and abstract notation to represent quantities one wants to manipulate to solve problems. Algebra 2 (known as Intermediate Algebra at the college level) builds on those concepts, but requires greater use of specific procedural formulas such as solving quadratic equations or manipulations of logarithmic functions.

“Algebra 2 for all students is a very unrealistic course, and always has been,” says Phil Daro, one of the authors of the Common Core State Standards for Math and former director of the California Math Project. “Real teachers can’t stand up there for weeks at a time with the majority of kids not understanding a thing. Teachers have had to accommodate reality in various ways. The name Algebra 2 is slapped on a wide variety of courses, students pass the courses, but the vast majority of college-going kids learn very little of what’s in the syllabus.”

Indeed, analyses of national tests as well as studies of high school transcripts have found a growing gap between the titles of courses and their content (Loveless, 2013). This partially explains large college remedial enrollments, even among students who pass high school math courses. California State University admits, for example, have passed Algebra 2 with at least a C, but a third don’t pass the placement exam for college-level courses, despite earning a mean high school GPA of 3.17.

Concerns about college readiness have led some states to require Algebra 2 for high school graduation. Minnesota and Connecticut, for example, have recently added it. National initiatives such as the American Diploma Project, led by the organization Achieve (which now runs one of the

new Common Core-aligned testing consortia), have promoted this emphasis to ensure more students have the math background that many colleges expect. By 2012, about three quarters of U.S. students, including a greater proportion of minority students, were taking Algebra 2 in high school (Loveless, 2013).

On the other hand, the argument that algebra-intensive requirements crowd out other courses that might be more beneficial for many students is gaining traction. In an analysis of the math demands of first-year community college courses, the National Center on Education and the Economy found that students need to use statistics, data analysis, and applied geometry—topics that are rarely taught in schools—more often than Algebra 2. To ensure that students acquire these skills, the report recommended ending the movement to require Algebra 2 for all students. “To require these courses in high school is to deny to many students the opportunity to graduate high school because they have not mastered a sequence of mathematics courses they will never need,” the report said.

A few states have also dropped the course from their high school graduation requirements for all students. These include Texas, which originated the movement to require Algebra 2. In 2013, the Texas legislature reversed the policy, allowing students

to opt out of Algebra 2 in favor of courses in statistics and algebraic reasoning. The move mainly affects students planning to attend community colleges (or no college), as most universities still require two years of algebra for admission. The situation is similar in California, where students need two years of math (not including Algebra 2) to graduate, but at least three years (through Algebra 2) to enter public universities.

**Common Core Implementation in K–12.** The Common Core standards being implemented in California and most other states retain the traditional emphasis on two years of algebra and a year of geometry in keeping with conventional standards. Yet, they represent a fresh approach to math content within traditional topic lists because of their emphasis on practice standards. These include math competencies such as problem solving, adaptive reasoning, and procedural fluency. The standards also acknowledge that certain high school topics are needed only by students pursuing STEM fields (see Box, *A Fresh Approach*).

Organized to promote what Schoenfeld calls “sense-making,” not just content knowledge, the new standards are designed to provide students with a stronger grounding in basic arithmetic and pre-algebra topics in the early grades to support eventual success with high school

content. They also constitute a break from the prior standards' reliance on discrete skills, in favor of a more holistic approach to conceptual understanding.

The standards were developed to address deficits of the previous standards, including the charge that they were “a mile wide and an inch deep.” In practice, though, when it came to high school mathematics, despite adding some probability and statistics standards, the authors kept most of Algebra 2 in order to ensure that the standards would be deemed rigorous.

Some observers are optimistic that these standards can lead to instructional improvements that will help more students proceed further in traditional math topics. Schoenfeld is among them. “If we make a significant investment in the mathematical understanding of teachers and in teacher preparation, a lot more people would get over the algebra hump than currently do,” he said. “Most kids would be able to go further in the math curriculum because it would make sense.”

There was interest among some developers of the standards in introducing a second pathway in the junior year similar to Texas' move, but time constraints and political realities limited their ability to make such major shifts. Instead, they attempted pragmatic reforms to Algebra 2, said Daro. “We tried to make it a more reasonable course for regular people. We did that. We could have done better. We were up against the same thing that everybody else trying to reform this is up against: the mentality that we need to know who to flunk,” he recalled.

**A Fresh Approach:** While the Common Core State Standards cover the traditional math topics at the high school level, they embrace the idea of alternative pathways in several ways:

- Notably, they include a set of “practice standards” that transcend math content and address specific capacities—such as problem solving, adaptive reasoning, conceptual understanding, and procedural fluency—that students need in order to have a mature grasp of mathematics.
- The standards are explicitly neutral on curriculum or course names, telling instructors that various pathways can be used to reach the outcomes. While reaching a certain benchmark on the tests will designate a student as ready to start college without taking remedial math, there are several versions of how the student can get there. The authors give two examples of curricular pathways for teaching the standards, and acknowledge room for others. The traditional sequence consists of Algebra 1 in ninth grade, Geometry in 10th grade, and Algebra 2 in 11th grade, while the integrated sequence weaves those topics together at each grade level.
- There are basic math standards, designed for all students, as well as an additional set of “plus” standards for students intending to pursue STEM majors or attend more selective universities.
- Though the standards call for four years of math, the assessments will be based on three years of material, so some students who are not interested in STEM fields or don't want to apply to selective colleges may opt out of the fourth year.
- Other students who wish to take Calculus or another college-level class while they are still in high school (for example, to boost their chances of admission to selective colleges or majors) can take a compressed or accelerated version of the curriculum, effectively covering five years' worth of material in four years.
- To deal with the difficulty that solving exponential equations presents for many students, the emphasis was placed on modeling with exponential functions, not on manipulating expressions.

Until the Common Core-based assessments are implemented, it will be hard to know what weight will ultimately be placed on specific topics versus competency in the practice standards. And it will take many more years before the success of Common Core will be known. Most observers say the standards will succeed in sparking significant change only if they are accompanied by robust investments in teacher training and professional development, and it is far from clear that these will materialize.

**Alternative Remedial Pathways in Community Colleges.** If the Common Core standards are successful in motivating improvements in math instruction, the need for college remedial courses could, in time, dramatically decrease even without changes in colleges' math instruction. However, some are skeptical that will happen, noting that similar claims were made when the 1989 math standards were introduced.

“That’s asking for a very long-term change in the whole culture of teaching and learning and our whole view of what it means to do math,” notes Stigler. “That’s a very long process.” Poor teaching has become so calcified in the traditional math pathway, he believes, that new standards will not be enough to change how teachers teach.

Even if the new standards succeed, it will take many years before they are fully integrated into K–12 schools, and students steeped in them begin enrolling in college. So, at least for now, remedial math education will remain a fixture of many public colleges. The idea of alternative pathways is gaining particular traction as a reform strategy among leaders of efforts to improve remedial education and thereby increase graduation rates at community colleges. Fueled by foundation support and policymaker concern about high failure rates, experimentation is taking place in states including Texas, Colorado, Georgia, Ohio, and California.

In many states, the highest level remedial course is Intermediate Algebra, which corresponds loosely to high school Algebra 2. The course is intended to prepare students for College Algebra, though neither course has a very precise definition. Intermediate Algebra courses are coming under scrutiny, especially given the backdrop of shifting undergraduate math requirements. As colleges continue to adopt alternatives to Calculus and College Algebra for their credit-bearing classes, some have begun to question the purpose of prerequisites like Intermediate Algebra.

In 2014, the American Mathematics Association of Two-Year Colleges (AMATYC) passed a resolution saying that Intermediate Algebra should

not be a “universal prerequisite for all college-level mathematics courses. College-level courses outside of the calculus-based course of study can be better served by other prerequisite courses that are more appropriate and relevant for preparing students for non-STEM courses of study. The content of a course, as defined by the course description and learning outcomes, should determine its mathematical level, prerequisites, and transferability.”

While acknowledging that an alternative like statistics, too, can be taught in an ineffective, procedural fashion, advocates think it stands a better chance of being taught well for several reasons. First, they say the challenge of changing the way teachers teach math may be easier to address with fresh content. Secondly, statistics is more immediately useful to students for things that interest them like analyzing sports data, following political polls, or understanding medical research. And for college students, it’s less likely to be a course that they took (and struggled with) in high school. Though there isn’t much evidence (aside from some very recent experiments that will be detailed in the second report), advocates are hopeful that new curricula will address the problem highlighted in 2003 by Carnevale and Desrochers:

Too many students get bogged down in the abstract procedures that remain the focus of much of the current mathematics curriculum. Others know the formulas and procedures but do not understand what they know well enough to use mathematics outside mathematics class.... Ultimately we will need a curriculum that teaches these higher level quantitative reasoning skills in a more applied and accessible context in which the goal is both knowledge and understanding.

The experiments are beginning to report surprisingly strong results. In its second year, working with 23 institutions in five states, the Carnegie Foundation's Statway program tripled remedial students' passage of college-level gateway math courses in half the time. An evaluation of eight early adopter colleges in the California Acceleration Project saw success in gatekeeper math courses improve more than fourfold compared to the standard sequence. A somewhat newer companion effort in Texas, the New Mathways Project, reports similar early findings, but its first evaluation was not yet available when this report was completed. Other approaches are being implemented in states including Georgia, Colorado, and Massachusetts. The promise of these experiments as well as obstacles to their success will be examined further in the second report in *Degrees of Freedom*.

### Diversification and Its Dilemmas

Given the traditional prominence of math requirements in the undergraduate curriculum, the implementation of alternatives faces a host of challenges. Some obstacles, such as tension over academic turf, may be political in nature. But others are more substantive, such as the economic need to encourage more students to pursue the STEM disciplines that rely on training in algebra and calculus. The prospect for scaling the alternative approaches, particularly at the pre-college level, will depend largely on how the dilemmas are resolved.

**Rigor and requirements.** For one thing, the alternative approaches face scrutiny about their level of rigor. There is little uniformity, especially when looking across educational segments. Some Introductory Statistics courses are actually quite advanced, assuming a background in calculus. These tend to be the type that more senior math professors are familiar with. But in fact, such courses constitute only 11 percent of introductory statistics enrollments (CBMS, 2010).

Technology use has made statistics more accessible by eliminating from introductory courses a great deal of the computation on which it is based. "Calculus lays the foundation of statistical theory, but the

majority of students don't need to understand calculus to understand Introductory Statistics," says Rebecca Nichols, Director of Education for the American Statistics Association. "They need to understand data, how to make decisions based on data, and what questions to ask before believing the results of a study."

Among AP courses, Statistics is treated as on a par with Calculus. For transfer from community colleges to public four-year universities in many states, Statistics is generally considered the equivalent of College Algebra. There is no data on how many Statistics courses—or which ones—require algebra, but there is increasing evidence from experiments with alternative curricula that many Algebra 2 topics are not required for students to learn statistics. "They need concepts from Algebra 2 to do statistics, but not necessarily the entire course," said Nichols.

A recent study at the City University of New York (CUNY) implied that Statistics students might not even require Algebra 1, or Elementary Algebra. Students who placed into elementary algebra were randomly assigned into one of two Elementary Algebra courses or into an Introductory Statistics class with a weekly workshop. The study found that most high school graduates who

tested into Elementary Algebra but skipped it were able to pass Statistics. It's unclear whether this proves that the students don't need even introductory algebra. It could also show that students retained enough of high school algebra to do well in Statistics, despite a low score on the placement exam.

While the CUNY finding heartened proponents of compressing students' remedial sequences to speed their entry into college-level courses, it also could make statistics vulnerable to being labeled insufficiently rigorous to be a college-level course. "They really do know that statistical material, but it's not based upon a strong understanding of mathematics," observed Wade Ellis, a retired community college mathematician.

The rigor question cuts both ways. More selective institutions that by definition cannot admit all students often use rigor as a screen to eliminate students who haven't passed difficult courses. Broad-access institutions like community colleges are in a different situation. Under pressure to increase graduation rates, many are focused on rooting out unnecessary barriers and reject the idea of rigor for its own sake. Given that some elite public institutions enroll large numbers of community college transfer students, there is an inherent tension between the two perspectives.

**Articulation and alignment.** A related challenge for alternatives is whether they will be accepted by four-year universities. Though some four-year institutions have embraced alternatives to College Algebra such as Statistics, most have been slower to consider alternatives to Algebra 2 (for entering freshmen) or Intermediate Algebra (as a remedial prerequisite for the math courses community college students can use for transferring).

Such policies present challenges to gathering evidence on the effectiveness of the alternative sequences. Despite their growth at community colleges, many college leaders are reluctant to expand the alternative remedial programs without assurance that their students can transfer smoothly. Challenges around alignment and transferability are the focus of the next report in this series.

Another dilemma is that the success, if it continues, of the remedial courses could call into question high schools' continued reliance on more traditional math courses. Higher education leaders have for years been advocating that K–12 schools focus more on college preparation, but now, just as K–12 reforms are moving beyond rhetoric to incorporate college readiness as an operating principle, higher education's definition of math readiness—on which those reforms have been based—is in flux.

The notion of eliminating Algebra 2 in high school appears to give educators the greatest pause. Years of effort have been expended ensuring that more students have access to the courses that are correlated with higher education opportunities. Access to algebra, in fact, has been considered a civil rights issue. This history leaves many cautious that alternative pathways would create separate but unequal tracks. "It's an interesting idea, fraught with danger," says Schoenfeld. "In the language of the 1989 math standards, the so-called 'non-college-intending' tracks all turned out to be dead ends."

UCLA's Green is only slightly less wary. "One of the problems this effort faces is that, on the one hand, the need for math keeps going up and getting more diverse, but at the same time, students' preparation for it may not be keeping pace," he noted. "You have a danger of people being limited throughout their lives by what math they got early on—or didn't. There's a lot of stuff that uses Algebra 2, and students who don't take it may be unaware that they are limiting their options later on."

"On the other hand," he acknowledges, "it's much better to have someone who genuinely understands modeling and quantitative reasoning and has a feeling for statistics than someone who took an Algebra 2 class but is totally bewildered by

it.” He believes that it’s incumbent on schools offering alternatives to inform students that such courses won’t prepare them for various scientific fields.

For now, a tentative consensus is emerging among some proponents of alternatives that high school is too early for students to opt out of Algebra 2. Thus, most of the experiments are occurring at the college remedial level. College students, after all, are in a position to know whether they will be pursuing a field that requires calculus. High school students, arguably, are not.

“I love Intermediate Algebra,” said Malcolm Adams, a University of Georgia mathematician who participated in his state’s task force to reform math curricula. “I believe everybody should see that at least once. I just don’t believe that everybody should be forced to be an expert at it.”

Nevertheless, it’s not hard to predict that if the early success of the community college alternatives is borne out in future experiments, discussion over alternatives in K–12 will only accelerate. Some observers expect future revisions of high school standards to incorporate the idea of a second pathway. “It’s too early to back off the Algebra 2 expectation for high schools,” said David Spence, president of the Southern Regional

Education Board, a regional association that promotes college readiness and other educational improvement initiatives in the South. “I think it will take four or five years for things to coalesce around some different pathways. The idea of less emphasis on the upper reaches of Algebra 2 for college readiness probably will creep into high schools.”

For now, K–12 leaders who think students need more grounding in math topics other than Algebra 2 are monitoring the outcomes of the community college experiments—plus a few taking place within K–12.

The Los Angeles Unified School District (LAUSD), for example, is collaborating with researchers at UCLA on a new data science class they describe as a “computation-based statistics and probability class.” Last fall, 10 teachers began the first pilots. According to the course description, the prerequisites are the same as for an Algebra 2 class, and the class will satisfy the math requirements for admission to UC and CSU in lieu of Algebra 2. Students who complete the course will be eligible to take AP Statistics. The course is part of a larger effort funded by NSF to expose more LAUSD students to statistics and computer science.

California’s state legislature is also growing impatient about students’

high school preparation. It recently passed two bills on high school computer science courses. One asks the public universities to develop guidelines for computer science courses to fulfill math requirements for admission. The other will allow districts requiring more than two math courses for graduation to count computer science courses toward graduation.

**Bridges and barriers.** Interestingly, both sides of the debate on alternatives are grounded in a concern about eliminating barriers for students: While proponents of alternative pathways view algebra-intensive curricula as a potential barrier to students’ success, critics fear that an education without advanced algebra itself constitutes a barrier.

Take the LAUSD data science course. There remains the possibility that some students taking it will discover belatedly that they want to become engineers or physicists and won’t have the necessary math preparation. That concern is one of the main arguments against alternatives. Proponents of alternatives say this risk isn’t sufficient reason to require all students to take the standard curriculum—or pre-emptively fail them.

“The sequence of abstract high school mathematics courses that prepares students for advanced degrees in mathematics and science is still crucial to our advanced economy,

but moving the entire school-age population through the academic hierarchy as a sorting strategy for producing elite mathematical talent required of a small share of college majors and fewer than 5 percent of the workforce does not match well with our more general needs for applied reasoning abilities and practical numeracy,” wrote Carnevale and Desrochers in 2003.

One solution is to make bridge courses—effectively remedial courses—available for students who need to make up material. Such programs are common in other fields: Students who discover after college that they want to become doctors, for example, can enroll in post-baccalaureate premedical programs. The Carnegie Foundation will design math bridge courses for community college students who take a statistics or quantitative reasoning pathway only to discover an interest in STEM. The idea also makes sense for community college students in associate degree or certificate programs who later discover they want to major, say, in math. Some universities that are addressing remediation challenges are also interested in the idea.

“It seems to me that would be a more constructive approach,” says Harry Hellenbrand, a provost at Cal State Northridge. “Considering the amount of money we waste by people flunking out of these remedial

courses, I would guess that if we put one tenth of that into preparing the bridge classes well, we would have more money for the math programs in general.”

## LOOKING AHEAD

As educational leaders look to improve high school graduation rates and streamline transfer, especially among minority and low-income students, math requirements are a critical lever. Whether students are applying to college, transferring from a two-year to a four-year university, choosing an undergraduate major or applying to medical school, math represents one of the greatest keys to success. Unfortunately, for too many students, the very courses intended to support that success are instead functioning as barriers.

Changing this will likely require a multi-faceted strategy that includes improvements to instruction at all levels. Indeed, every segment of public education in California—from K–12 schools to community colleges to the two university systems—is engaging in innovations to ensure that students have essential math preparation. For now, however, each is pursuing these innovations largely in isolation. With little understanding of what the others are doing, the inevitable tensions among them are not transparent, leaving insufficient clarity about what preparation is

needed and by whom. Curriculum standards and university admission requirements often stand in as means of communication across the educational segments.

Making sure students are successful in required math courses entails improving how those courses are taught, but it also involves making sure the requirements are appropriate, accurately communicated, sufficiently aligned across segments, and relevantly assessed. As reports two and three in this series will discuss, California has much work to do in this area to ensure that math requirements can support student success and advancement.

Undergirding the series of reports are a few assumptions. While there is unquestionably a need to improve instruction beginning in elementary school, as the Common Core State Standards are designed to do, institutions of higher education cannot afford to wait a dozen years until cohorts of students fully exposed to the new standards reach their doors. California’s schools and colleges need to consider practical approaches that will best serve the students they have now, while simultaneously investing in and evaluating longer-term strategies.

The second and third reports in this series will include specific recommendations about policies and

practices that do this. This report concludes with a set of principles that will underscore those specific recommendations.

New policies and practices should be driven by several principles:

- Keep the focus on maximizing student success statewide;
- At the college level, base remedial policies and other requirements on equipping students for subsequent coursework, career, and citizenship (rather than abstract notions about desired knowledge);
- Increase communication so segments can better understand the implications of their policies on students in other segments and minimize misalignment;
- Apply greater scrutiny to academic requirements noted for high failure rates; and
- Make room for experimentation and innovation while gathering evidence.

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### **About the Author**

Pamela Burdman is a Berkeley-based policy analyst on college access, readiness, and success and a former program officer at the William and Flora Hewlett Foundation. She previously served as a journalist. Her writing has appeared in the *San Francisco Chronicle*, *The New York Times*, *San Jose Mercury News*, and other publications.

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Policy Analysis for California Education is an independent, non-partisan research center based at Stanford University, the University of California-Davis, and the University of Southern California. PACE seeks to define and sustain a long-term strategy for comprehensive policy reform and continuous improvement in performance at all levels of California's education system, from early childhood to post-secondary education and training. PACE bridges the gap between research and policy, working with scholars from California's leading universities and with state and local policymakers to increase the impact of academic research on educational policy in California.

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LearningWorks aims to strengthen student achievement in community colleges. It does so by facilitating, disseminating and funding practitioner-informed recommendations for changes at the system and classroom levels, infusing these strategies with statewide and national insights. LearningWorks has been supported by the William and Flora Hewlett Foundation, the Walter S. Johnson Foundation and the James Irvine Foundation.

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